

PRACTICAL PHYSICS

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P R E F A C E

THE importance of experimental work in the teaching of Natural Science is now well recognised. Some experimental acquaintance with the phenomena which are being studied is almost indispensable for their proper understanding, and no one would now attempt to teach Physics as a mere "book-work" subject. Practical Physics, however, has a value apart from its function of providing a basis for theoretical teaching. It provides one of the best means of educating the student's powers of accurate observation and accurate reasoning.

For the training to be of real value, however, the observation must be accurate, and the reasoning correct. This accuracy is of greatest importance, just where it is most difficult to obtain it, namely, in the first year's course in the subject. Beginners are only too apt to assume that accuracy is bound up with elaborate and costly apparatus, and is not to be expected from the simple means at their disposal. They thus form habits of slovenly and inaccurate working which it is exceedingly difficult to eradicate afterwards. In Practical Physics as in other things "well begun is half done," and the student who has worked conscientiously and intelligently through the course outlined in the following pages will find few difficulties to be overcome in the remainder of his work in Practical Physics.

The course has been designed to cover the practical work in Physics which is usually demanded of a student in the first medical examination, or in an examination for a school certificate. The book thus forms, in some measure, a companion

volume to the author's *Manual of Physics*. It is, however, except in scope, quite independent of the latter, and I have not hesitated to vary both the order and the treatment of the different subjects where it seemed advantageous from a laboratory point of view.

I had not thought it possible, when I undertook the writing of this volume, that so many words would be required to describe the simple experiments contained in it. Accuracy in Practical Physics, however, is largely a matter of careful attention to small details, and even small details may take long to describe. Much experience as a teacher and an examiner has taught me that no precautions are obvious to the average student. It is only by training and by experience that the arts of practical measurement in Physics are acquired. I have, therefore, described at least one experiment of each class in very minute detail, and have attempted at the same time to explain the why and the wherefore of each of the precautions described. Some attempt has also been made to indicate the order of accuracy to be expected from the different kind of measurements made in the course of the experiment, and to impress upon the student the importance of adjusting the various factors involved so as to make the final result as accurate as possible. All these considerations form an integral part of the art of experimenting, and cannot be introduced too early into the course, but they undoubtedly take up space in describing. From considerations of space, however, these details are not usually repeated when a second experiment of a similar type is described; the student is simply referred back to the previous experiment in which they have been described in detail. In this way it is hoped that the volume will supply all the information the student needs, without at the same time eliminating all necessity for active thought on his part. Students often acquire a considerable mechanical and unintelligent facility in carrying out detailed printed instructions—a facility which, though it may have its uses, is of little educational value.

The various divisions of the subject are arranged in the

usual order, but it is not necessary, or perhaps even advisable, that they should be taught in this order. Practical Mechanics and Practical Electricity are usually found to present more difficulties to the beginner than Heat and Optics. To whichever section is taken first, however, the preliminary chapter on Practical Measurements forms a suitable and almost indispensable introduction.

The range of the experiments has been limited by the schedules of the examinations for which the volume has been primarily prepared. Even so, the selection of experiments from the large number which might have been described, has been one of no little difficulty. With a few exceptions, experiments of a merely qualitative kind have been omitted. The great majority of the experiments included are capable of giving results which are accurate to two or three per cent. As the volume is intended for the use of the student, I have also ruled out experiments which, either from the manipulative skill required, or from the complexity or cost of the necessary apparatus, seemed more suitable for the lecture table than the elementary laboratory.

The apparatus required is usually of a simple and fairly inexpensive type. Elaborate and expensive apparatus is neither necessary nor desirable for students beginning the subject, and much of the apparatus described in the book could well be made by the students themselves with a little assistance.

The student should not leave any experiment until it has been carried to a successful conclusion. One experiment thoroughly mastered is of more value than a large number carried out in an inaccurate or slovenly manner. In my own classes I have always insisted on seeing a correctly written up account of the previous experiment before allotting a fresh one. As a result, however, it is almost inevitable that students will progress at very different rates. In order that the class may be kept more or less together, which is often desirable, I have added some additional exercises at the end of each chapter. These have mainly been selected from examination papers

and may be used as tests of the previous work. A student who has mastered the previous experiments in the section should have no difficulty in working them through successfully without further instruction.

Many of the diagrams have been drawn especially for this book to illustrate the principle of the apparatus rather than its appearance. A few diagrams have been reprinted from the author's *Manual of Physics*. I am indebted to Messrs. F. E. Becker & Co. (London) and Messrs. W. G. Pye & Co. (Cambridge) for their kindness in supplying illustrations of some of their apparatus.

J. A. C.

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FOREWORD TO THE STUDENT

TO ERR IS HUMAN. A recognition of this fact is the beginning of wisdom in Practical Physics. Liability to error can, however, be greatly reduced by strict concentration on the work in hand, and by careful and methodical methods of working. Possible sources of error in different kinds of measurement are fully discussed under the different experiments described in the following pages, and deserve attention. The following general rules will also be of service.

Before commencing an experiment make quite sure that you understand the principle of the experiment, and the measurements which will have to be made to obtain the required result. It is unpleasant to realise, after the experiment has been completed and the apparatus put away, that some quite essential measurement has been overlooked.

In most experiments measurements are made by reading the position of some particular point on the scale of some instrument. Make quite sure that you understand the scale on the instrument before you attempt to use it. The units in which the instrument reads should be marked upon it. If they are not, consult some one in authority. Then examine the scale closely to make sure that you understand the way in which it is graduated. Different instruments are graduated in different ways. Thus if your reading is three scale divisions beyond the division numbered 10 the reading might be 10.3, or it might be 13. It might even be 10.6 because some scales have the units subdivided into fifths. These points can be determined by examining carefully the scale itself.

As soon as the measurement has been made record it in your notebook (not on a scrap of paper) and record also the

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units in which it is made. Trust nothing to your memory; it may deceive you. Record also exactly what it was you measured. If you measure the diameter of a wire, put down "diameter of wire." This will avoid the uncertainty, which is otherwise sure to arise later, as to whether the number represents the diameter or the radius of the wire. If the nature of the measurement permits it, always verify your observation by a second reading of the scale.

Always record the actual observations made and not deductions from them. If you find the diameter of a wire to be 2.6 mm., write down "diameter of wire = 2.6 mm.," and not radius of wire = 1.3 mm. Mental arithmetic is a useful form of mental gymnastics but it is too dangerous to be practised in a Physical Laboratory. It is possible to make a mistake even in dividing by two. If the actual measurements are available the mistake may be discovered on recalculation. If only the erroneous deduction is preserved correction becomes impossible.

When the measurement has been made and recorded always consider whether the recorded result is a possible one. The diameter of a cylinder as thick as your finger is obviously not 1.5 mm. It may be 1.5 cm. Mistakes can often be avoided in this way by the use of a little common sense.

Accustom yourself to make your laboratory record sufficiently full and clear, so that a fellow-student, who had not been present at your measurements, would be able to calculate the result from your observations. The correct way of displaying the measurements is indicated in the examples which follow some of the experiments described. The result should always be calculated from the original record. It is an excellent plan to keep a separate notebook in which to write a full description of the experiment and the theory underlying it, and the experimental results may be transcribed into this to complete the record. It is, however, more difficult than is generally supposed to make a correct copy of a series of figures, and the fair copy has never the same value as the original laboratory record.

FOREWORD TO THE STUDENT

The evaluation of the result will generally involve some arithmetic, and experience shows that errors in arithmetic are even more common than errors in observation. Here, again, a little common sense will save many mistakes. It is, in fact, generally worth while to make a rough calculation so as to obtain an approximate value for the result, before proceeding to the exact evaluation. Thus before working out the fraction

$$\frac{59.3 \times 17.84}{30.75}$$

it will be an advantage if the student first works

out some approximately equal expression such as $\frac{60 \times 18}{30}$

the value of which is obviously 36. The correct result is, therefore, somewhere about 36. The more exact calculation shows that it is actually 34.40. This will at once enable the student to detect any glaring errors in the arithmetic, and also give him the correct position for the decimal point. The difference between 34.40 cm. and 344.0 cm. is of more practical importance than some students seem to realise.

All quantities of the same kind must be reduced to the same units before being used in the calculation. Thus if some of the lengths measured in an experiment are in centimetres and others in millimetres, the latter must be converted into centimetres before they are allowed to enter into the calculation.

Tables of logarithms, antilogarithms, reciprocals, etc., are a great aid to rapid calculation if the student is sufficiently acquainted with them to use them with facility and accuracy. Four-figure tables, which give the result of the calculation to an accuracy of about 1 in 1000, will be sufficient for the experiments in the present volume, and form a necessary part of the student's equipment. A 10-inch slide rule, giving an accuracy of about 1 in 500, may also be employed with advantage.

There is no such thing as the "answer" to a physical investigation. No two observers with the same apparatus would obtain exactly the same results. If, however, they were both competent their results would only differ by a small

amount, which may be regarded as the error inevitable to the given apparatus and experiment. A correct result is one which approximates, within the limits of experimental error of the apparatus used, to the results of the best observers. It should be the object of the experimenter to obtain, by careful and intelligent work, a result as nearly correct as the means at his disposal will allow.

An experiment should never be left until it has been completely mastered and both the theory and practice have been completely grasped. Most experiments involve several distinct kinds of measurement, and often several physical principles, and one such experiment well done will add more materially to the student's attainments in the subject than many experiments inaccurately performed and imperfectly understood.

Apparatus should always be treated with care, and should be left in a clean and tidy condition.

Finally the student may be advised to follow closely the directions given for performing the experiments, at any rate until he has advanced sufficiently in the art to be able to improve upon them. Success in Practical Physics is largely a matter of attention to details, and a single error may be sufficient to spoil a large amount of otherwise useful work. On the other hand, a student who by careful application has succeeded in carrying out successfully even a part of the work described in the following chapters will find that in so doing he has made a by no means inconsiderable addition to his knowledge and his capabilities.

PRACTICAL PHYSICS

PRACTICAL MEASUREMENTS

§ 1. MEASUREMENT OF LENGTH

THE methods adopted for measuring length depend on the magnitude of the length to be measured and the accuracy which it is desired to attain. For lengths greater than 10 cm. the measurement can generally be made with sufficient accuracy by means of a scale divided into centimetres and millimetres. These scales are generally made either of steel or of boxwood. The former are the more accurate, and are generally preferred when the scale is a short one (say less than 30 cm.). For greater lengths, scales made of boxwood are generally used, as a steel scale 1 metre long would either be very heavy or too pliable for convenience. For lengths greater than 1 metre measuring tapes are generally used, arranged so that they can be wound on a reel when not in use.

Scales graduated in inches and fractions of an inch can also be used. These are usually subdivided into eighths of an inch, which leads to unnecessary arithmetical complications. A scale divided into tenths of an inch is to be preferred for scientific work if an inch scale must be used. Scientific measurements are, however, usually made in the metric system. This system, in which the centimetre is the unit of length, is not only the most convenient, but has the further advantage that it is international, and has been adopted for scientific work in all countries.

Measurement of a length by means of a scale is a comparatively simple operation, but care must be used if the result is to be trustworthy. The following experiment, which may be regarded as a simple exercise in the measurement of length by

a scale, will enable us to determine the relation between the two kinds of units :

EXPERIMENT 1.—To measure a length (a) in centimetres, (b) in inches, and to deduce the number of centimetres equivalent to one inch.

Rule a pencil line about 10 inches long on a sheet of paper, and with a finely pointed pencil make two pencil marks at right angles to the line, one at each end. The object of the experiment is to determine the distance between these two marks. Lay a centimetre scale along the ruled lines so that the graduations are in contact with the line. If the scale is a thick one and not bevelled, it should be held upright so that the divisions are in contact with the paper. It is quite impossible to make an accurate measurement unless the object to be measured is in contact with the graduations on the scale. As the end of the scale is liable to be worn down or otherwise damaged, adjust the scale so that the 1 cm. mark is in exact coincidence with one of the two cross lines on the paper, and then, without moving the scale, take as accurately as possible the reading of the other cross line on the scale. The second mark will probably not coincide exactly with one of the graduations of the scale. The small distance by which it overlaps the exact division should be estimated by subdividing the division into which it falls into ten equal parts, and noting with which of these imaginary subdivisions the mark coincides. With a little practice this estimation by eye, as it is called, can be carried out accurately to one-tenth of a division. Having recorded the exact reading, note again the reading of the other mark, to make sure that the scale has not moved. The difference between the two readings will give the distance between the marks in centimetres.

Repeat the observations, using an inch rule. The observations may then be calculated as follows :

	Cm.	Inches.
Reading of second mark	21.46	9.06
Reading of first mark	1.00	1.00
Distance between marks	20.46	8.06

$$\therefore 8.06 \text{ inches} = 20.46 \text{ cm.}$$

$$1 \text{ inch} = 2.538 \text{ ,,}$$

The correct value is 1 inch = 2.540 cm. The error in

measurement in this example is therefore rather less than 1 in 1000.

The measurement of a length is often facilitated by the use of a pair of dividers or calipers. A very useful pattern is shown in Fig. 1. One of the legs, B, moves over a circular bar which is rigidly fastened to the leg A. The end A is placed in exact coincidence with one end of the length to be measured, the other end B is opened until it coincides with the other end of the length, and is then firmly clamped in

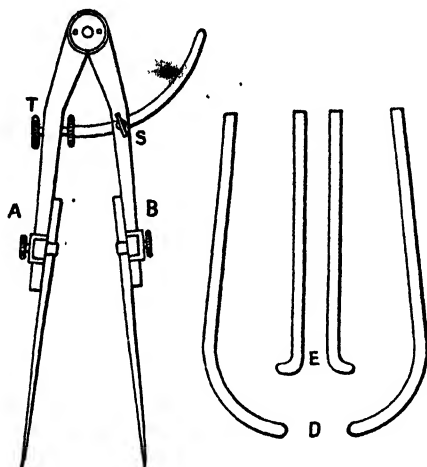


FIG. 1.—Calipers.

position on the bar by means of a clamping screw S. In some forms of instrument the adjustment is facilitated by a small tangent screw T attached to the bar which allows A to be moved very slowly through a short distance when B is clamped. Points of different shape for different kinds of measurements are provided. Points such as D (Fig. 1) will enable us to measure the outside length of a solid such as a cylinder or a sphere, while with points such as E we can measure the inside diameter of a fairly wide tube. Dividers should always be used when the distance or length to be measured is such that we cannot bring both ends simultaneously on to the scale.

EXPERIMENT 2.—To measure the diameter of a cylinder by calipers.

Using points curving inwards open the calipers until they slide easily over the cylinder when held at right angles to its length. Then gradually close the legs until the cylinder can be felt to be just touching both points simultaneously when it passes through them. That is to say, the cylinder can just be moved through the points without using force. Now lay the points on the scale so that the innermost point of one coincides exactly with some graduation on the scale, and read off the position of the innermost point of the other. The difference between these two readings is the shortest distance between the two points, and is obviously equal to the diameter of the cylinder.

THE VERNIER AND VERNIER CALIPERS

Instead of estimating fractions of the smallest division of the scale by eye we can use a device known as a vernier, which will enable us to make the division with greater certainty and accuracy.

EXPERIMENT 3.—To illustrate the principle of the vernier.

The principle of the vernier will be illustrated more conveniently if we work on a fairly large scale. Take a scale divided into centimetres. (If the scale is also subdivided into millimetres these subdivisions can be ignored.) Take a piece of fairly stiff cardboard having one straight edge, and laying the straight edge along the scale mark off a distance AB (Fig. 2*a*) of exactly 9 centimetres on the card by two short fine pencil lines. Divide the distance between these lines into ten equal parts. The ordinary geometrical method of doing this is to draw through one of the points, A, a straight line making a convenient angle with AB, and to step off along this line 10 equal parts by means of a pair of dividers with sharp points. The last of the points so obtained is joined to B by a straight line B*b*, and straight lines are drawn to AB through the remaining points parallel to B*b*. AB is then divided into ten equal parts. Take A as the zero of the vernier and cut off the card square at division A. The vernier is now complete.

To use it to measure, say the length of a rod, lay the rod along a centimetre scale so that one end of the rod coincides

with the zero of the scale. Let us suppose that the other end of the rod lies between the ninth and tenth division of the scale so that the rod is rather more than 9 cm. long (as in Fig. 2*b*). Bring the zero end of the vernier into contact with this end of the rod, the vernier scale lying in contact with the centimetre scale. Note which graduation on the vernier coincides most nearly with an actual centimetre graduation on the main scale. Suppose that coincidence occurs with graduation 4 on the vernier. Then the length of the rod is 9.4 cm.

The distance between two successive marks on the vernier is nine-tenths of a centimetre. The difference between one division on the scale and one division on the vernier is thus

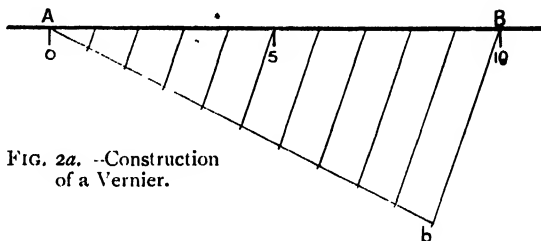


FIG. 2*a*.—Construction of a Vernier.

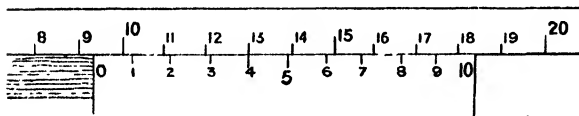


FIG. 2*b*.—Use of a Vernier for Measurement of Length.

one-tenth of a centimetre, or one millimetre. Now start at the mark 4 on the vernier which coincides with a graduation on the scale, and count backwards. Mark 3 on the vernier is thus 1 mm. beyond the next graduation on the scale, mark 2 is therefore 2 mm. beyond the corresponding graduation, and so on. Thus the zero of the vernier scale is 4 mm. beyond the 9 cm. graduation on the main scale, or the distance by which the rod overlaps the 9 cm. graduation is 4 mm. Thus to read the vernier we have merely to note the division on the vernier which coincides most nearly with a division on the main scale. If the vernier is constructed, as in this experiment, so that ten vernier graduations correspond to nine scale divisions, the number on the vernier division

will give us the next decimal place, or in other words it will enable us to divide each scale division into tenths.

In practice, of course, the vernier is always applied to the smallest graduations on the main scale. Thus a scale divided into cm. and mm. would have a vernier constructed so that ten vernier divisions were equal in length to 9 mm. The vernier would then read in tenths of a millimetre. If the vernier were made so that twenty vernier divisions were equal to 19 mm. the vernier would read in twentieths of a millimetre. The proof of this is left to the student as an exercise.

The principle of the vernier is applied in the Vernier Calipers. These are shown in Fig. 3. The bar carrying the graduated scale has a cross-piece rigidly attached at one end, while a second cross-piece carrying the vernier scale slides

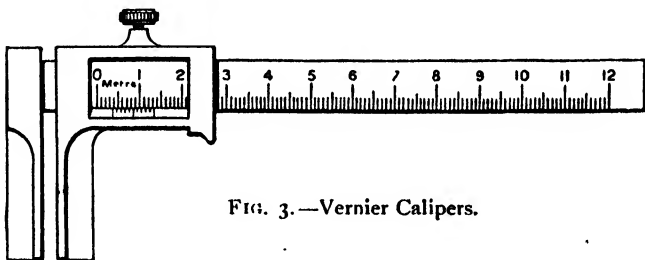


FIG. 3.—Vernier Calipers.

along the bar, and can usually be clamped in any required position by a clamping screw. When the sliding piece is pushed up so that the two jaws are in contact the zero of the vernier should coincide with the zero of the scale. If now the jaws are opened and some object placed between them, and the jaws again closed down upon it, the reading of the scale will obviously give the thickness of the object.

EXPERIMENT 4.—To measure the diameter of a circular coin by the vernier calipers.

The vernier is first examined to ascertain whether it reads in tenths or twentieths of the smallest scale division, and the calipers are then closed, and the reading taken. If the calipers are accurately set, the zero of the vernier will coincide with the zero of the scale. If not, take the actual reading of the vernier scale. Suppose, for example, that the second division of the vernier coincides with a scale division. The

actual reading when the jaws are closed is then 0.2 mm. or 0.02 cm., and this must be subtracted from subsequent readings to arrive at the true distance apart of the jaws. Open the calipers and place the coin, say one penny, with its face parallel to the slide, and close down the jaws upon it, gently, but firmly. Again read the scale by means of the vernier and subtract the "zero" reading from it. The difference will give the diameter of the coin correct to one-tenth of a millimetre. Several different diameters should be measured in this way and the mean taken.

THE SPHEROMETER AND THE SCREW GAUGE

The vernier calipers will give a measurement correct to one-tenth or one-twentieth of a millimetre, according to the vernier scale, but if the distance to be measured is itself very small, this may still leave room for an appreciable percentage error. If we were measuring the diameter of a wire 1 mm. in thickness, the error involved might be as large as 10 per cent. For very small distances we need some more delicate method of subdividing our smallest scale division, and this is done most conveniently by applying the principle of the screw. If a screw moving in a fixed nut is turned through one complete revolution it will move forward a distance equal to the distance between two successive threads on the screw. This is known as the *pitch* of the screw. If the screw has a pitch of $\frac{1}{2}$ mm. one complete revolution will carry the end of the screw forwards or backwards through $\frac{1}{2}$ mm., and turning the screw head through $\frac{1}{100}$ th of a revolution would move the screw through $\frac{1}{100}$ th of $\frac{1}{2}$ mm. or $\frac{1}{200}$ th mm. This principle is applied in two different instruments, the spherometer and the screw gauge.

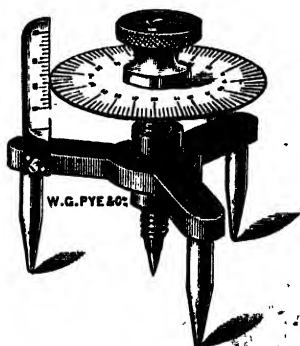


FIG. 4.—The Spherometer.

The spherometer (Fig. 4) consists of a little platform standing on three sharp points, and carrying a fixed nut, in which

middle. A screw of small pitch works in this nut, and a large circular metal plate is firmly attached to the screw-head. This disc is divided along the edge into a convenient number of parts, generally 100. A vertical scale is attached to the platform. This serves as an index, and also to measure the total number of turns of the head.

EXPERIMENT 5.—To measure the thickness of a glass slip with the spherometer.

Ascertain the scale of the instrument. This is important, as there are a good many different ways of graduating spherometers in use, and the scale is not always specified on the instrument. Find, first, whether the vertical scale reads in terms of fractions of an inch. If this information is not engraved on the instrument consult some one in authority. Then determine the number of revolutions of the screw-head required to move the screw forward through one of these divisions. Suppose that the divisions on the vertical scale are millimetres, that the screw-head is divided into 100 parts, and that two complete revolutions of the screw carry it forward one scale division, *i.e.* 1 mm. Then each division of the head is equivalent to a forward movement of the screw of $\frac{1}{200}$ th of a mm. or 0.005 mm.

Now place the spherometer on a plane sheet of plate glass (usually supplied with the instrument) and screw down the screw until it is just in contact with the plate. This can be ascertained by gently flicking one of the legs of the instrument. If the centre screw is not in contact with the glass the instrument will slide as a whole. If the centre screw is touching it will rotate round the screw. The exact point when contact is made can thus be determined. Take the reading first of the position of the head on the vertical scale, and then of the division on the head which comes against the edge of the vertical scale. As the head makes two revolutions for 1 mm. it will be necessary to notice whether it has or has not completed one revolution since passing the lower of the two graduations between which it is resting. If it has we must obviously add $\frac{1}{2}$ mm. to the reading. Thus if the head is between the 2 and 3 mm. graduations of the vertical scale and the division opposite the scale is 22, then if the head requires less than one complete revolution to bring it down to the 2 mm. mark,

the reading is $2 + (22 \times 0.005) = 2.110$ mm.; if more than a complete revolution is required to bring the head down to the 2 mm. mark, the reading is $2 + 0.5 + (22 \times 0.005) = 2.610$ mm. The scale is a little awkward to use, but it is the one generally adopted by instrument makers for spherometers.

Now place the cover slip (or other object) on the glass plate and adjust the screw, so that when the three legs of the spherometer are standing on the glass plate the point of the screw is just in contact with the top of the slip. Again take the reading. The difference in the two readings gives the thickness of the cover slip.

The screw gauge (Fig. 5) consists of a metal frame into one end of which is fixed a steel plug A, with an accurately planed

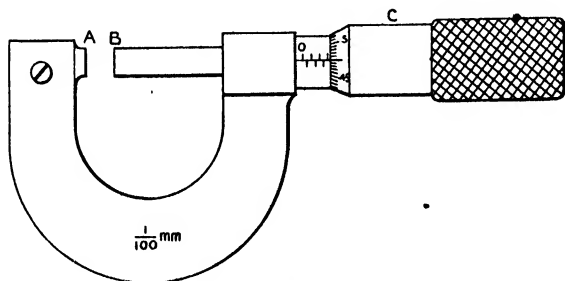


FIG. 5.—The Screw Gauge.

face, while the other end carries the nut through which the screw turns. The end B of the screw is planed so as to be accurately parallel to the face of A. A scale of millimetres and half-millimetres is engraved on the frame of the instrument, and the screw-head is prolonged into a hollow cylinder C which moves over this scale. This screw-head is divided into 50 equal parts, each of which, therefore, represents $\frac{1}{50}$ th of a millimetre. The line on which the scale is engraved serves as the index for the scale on the screw-head.

EXPERIMENT 6.—To measure the diameter of a wire by the screw gauge.

The screw is turned so that the two planed faces are in contact. The screw must never be forced, as, owing to the large mechanical advantage given by a screw it is quite possible

to exert a sufficient translational force actually to strain the framework of the instrument. The reading is then taken, and will be found to be approximately zero, that is, the zero of the screw-head will approximately coincide with the line of the scale. Take the actual reading and note whether it is positive or negative. If the screw-head is between the zero and the first division of the scale, and graduation 3 on the screw-head is against the index line, the reading is 0.03 mm., and this amount will have to be subtracted from succeeding readings. If, however, the head is on the other side of the zero, and if, say, division 47 coincides with the mark, the reading is -0.03 mm., and 0.03 mm. must be added to subsequent readings. The zero error, as it is called, will be a constant for the instrument, and need only be taken once during each set of measurements.

The wire is smoothed out to remove kinks, and straightened, and is then placed between the two faces A and B, and the screw is turned until the wire is just gripped between them. The reading is again taken. The difference between the two readings gives the diameter of the wire.

Example:

Reading of screw gauge when closed = +0.024 mm. (estimating
tenths of a division on the
screw-head by eye)

Reading with wire between jaws = 1.5 mm. (on scale)
+ 0.276 mm. (on screw-
head)

= 1.776 mm.

Diameter of wire = 1.776 - 0.024

= 1.752 mm.

The last figure will be somewhat doubtful, and will depend on the force with which the screw-head is turned. It is, however, worth while to try and estimate it.

A second diameter at right angles to the first is then measured in the same way, and the operations repeated at three or four different points on the wire.

§ 2. MEASUREMENT OF CURVES

THE measurement of the length of a curved line, such as the circumference of a circle or the perimeter of a curved solid, cannot be performed with the same accuracy as the measurement of the length of a straight line. Fortunately measurements of this kind are not of frequent occurrence in practical work. The measurement of the length of a curved line on paper is most conveniently performed by means of a little instrument used for map measuring, where the problem is one of importance. This instrument consists of a small fine-toothed wheel which revolves like a nut, on a fixed screw (Fig. 6), so that as the wheel rotates it moves backwards or

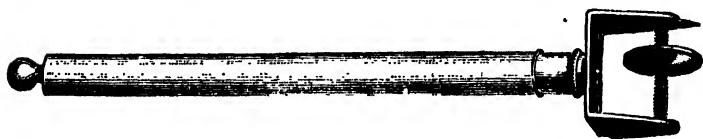


FIG. 6.—The Opisometer, or Curve Measurer.

forwards along the screw. The instrument is sometimes dignified with the title of opisometer. The wheel is turned until it rests against one end of the screw, and is then placed on one end of the curve to be measured and moved along it in contact with the paper, the wheel following out the contour of the curve. On reaching the end of the curve the wheel is lifted. It is placed on the zero mark of an ordinary straight scale and run over the scale in the opposite direction until the wheel is back again in its original position at the end of the screw. The distance moved along the scale is obviously equal to the length of the curve.

The length of a curve can also be measured by aid of a piece of thread. Place one end of the thread at one extremity of the curve, and arrange a short piece of the thread so that it lies exactly along the curve. Fix this down by pressing on

it with the nail of the forefinger; adjust a second short length of the string along the next portion of the curve in the same way, and continue until the end of the curve is reached. Cut the thread exactly at the end of the curve, and measure the length of the thread by placing it over a scale, being careful not to stretch it. Its length will be equal to the length of the curve.

The circumference of a cylinder can be measured by winding a thin thread round it say ten times and cutting the thread so that there are an *exact* number of turns, *i.e.* the two ends of the thread are exactly in line. (The turns should of course lie evenly side by side, and should not cross or overlap each other.) Unwind the thread and measure its length with a scale.

Alternately, wind a strip of paper round the cylinder so that its ends overlap and prick through the overlap with a fine needle so as to make a mark on each layer of the paper. Unwind the strip and measure the distance between the pin pricks by means of sharp-pointed dividers and a scale.

These experiments are illustrative only. In practice if we desired to know the circumference of a circular cylinder we should measure its diameter, d , by sliding calipers, or screw gauge, according to its size, an operation which can be performed with accuracy. The circumference would then be deduced from the relation that the circumference of a circle is equal to πd . The method might be required, however, for finding the perimeter of a solid which had no easily recognisable geometrical form. Exercises on these measurements will be found in the examples at the end of Book I.

§ 3. MEASUREMENT OF AREAS

If the figure is bounded by straight lines its area can be determined most accurately by dividing it up into a number of rectangles or triangles. The area of a rectangle is the product of its length into its breadth, while the area of a triangle is the product of the base into half the height. Similarly the area of a circle would be obtained by measuring its diameter, d , and using the relation :

$$\text{Area of a circle} = \frac{1}{4}\pi d^2.$$

The area of a figure bounded by irregular outlines is determined most conveniently by an instrument known as a planimeter. This is not likely to be available in an elementary laboratory. Two elementary methods are available, either of which will give a good result with a little care.

EXPERIMENT 7.—To determine the area of a figure by means of squared paper.

The figure is transferred to squared paper, *i.e.* paper printed with two sets of equidistant parallel lines at right angles to each other. These lines are usually either 1 mm. or $\frac{1}{10}$ th inch apart, so that the surface of the paper is divided into little squares each of which has an area of either 1 sq. mm. or $\frac{1}{100}$ th sq. inch, according to the ruling. Every tenth line is generally printed a little darker than the rest, so that the surface is divided by these heavier lines into squares of area either 1 sq. cm. or 1 sq. inch. The area of the figure can thus be found by counting the number of squares which it contains.

This must be done systematically if no squares are to be missed and none counted twice. First count the number of complete large squares contained within the boundaries of the figure, marking each square with a pencilled cross as it is counted. Note down the number. Then count the number of small squares left over, putting a dot in each as it is counted. It will save trouble if the uncounted area is first divided into convenient portions, and the number of small squares in each portion counted, and recorded separately. At the boundaries

of the figure where the boundary will cut across some of the squares the portions of squares which appear greater than half a square are counted as whole squares, while those which are less than half a square are neglected.

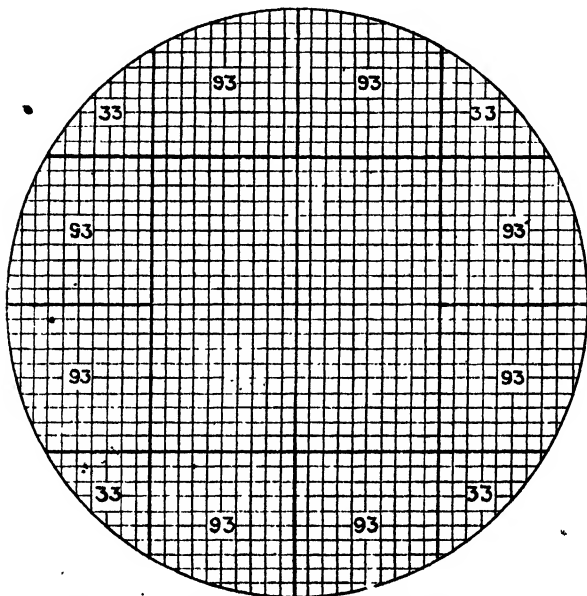


FIG. 7.—Measurement of the Area of a Circle, by use of Squared Paper.

As an exercise draw on inch graph paper a circle of 2 inch radius. Determine the area of the circle by counting the squares, and compare the result with that given by the formula, $\frac{1}{4}\pi d^2$.

Example :

Circle of 2 inch radius drawn on inch graph paper

1 small square = $\frac{1}{100}$ th inch

Number of complete large squares = 4

Number of small squares (see Fig. 7) = $8 \times 93 = 744$

$4 \times 33 = 132$

876

Area of circle = $4.00 + 8.76$ sq. inches = 12.76 sq. inches.

$\frac{1}{4}\pi d^2 = \frac{1}{4} \times 3.1416 \times 4^2 = 12.57$ sq. inches.

The result would have been more accurate if smaller squares, e.g. millimetre squares, had been employed.

EXPERIMENT 8.—Measurement of area by weighing.

If a sheet of cardboard or metal foil of uniform thickness and material is taken, the weight of any portion of the sheet will be proportional to the area. Flatten out a piece of thin sheet zinc (tin-foil, or thick cardboard may be used instead) and transfer to it the figure whose area is required. Cut out the figure carefully with a pair of sharp scissors, and weigh the figure so obtained. On the same sheet draw a figure, such as a square or rectangle, the area of which can be obtained by measuring its dimensions. Cut out this figure also, and weigh it. The areas of the two figures are in the same ratio as their weights.

Example :

Circle of radius, 4 cm., transferred to thin sheet zinc.

Weight of figure = 3.12 gm.

Weight of square of side 10 cm. = 6.25 gm.

$$\frac{\text{Area of circle}}{\text{Area of square}} = \frac{3.12}{6.25} \quad \therefore = 0.499$$

But Area of square = $10 \times 10 = 100$ sq. cm.

\therefore Area of circle = $100 \times 0.499 = 49.9$ sq. cm.

Verification.— $\frac{1}{4}\pi d^2 = \frac{1}{4} \times 3.1416 \times 8^2 = 50.26$ sq. cm.

The error in this method lies partly in the uncertainty as to the perfect uniformity of the sheet, partly in the difficulty of exactly following the outlines of the figure.

Either of these methods might be used to obtain an experimental value for the constant π . Thus taking the figures in the last example we have

$$\begin{aligned} \text{Area of circle (by measurement)} &= 49.9 \text{ sq. cm.} = \frac{1}{4}\pi d^2 = \frac{1}{4}\pi 8^2 \\ 16\pi &= 49.9 \\ \pi &= 3.12 \end{aligned}$$

The experiment in this form is merely set as a test of accurate experimentation. The value of π is, of course, actually obtained from mathematical calculations.

§ 4. MEASUREMENT OF MASS

THE mass of a body is measured by comparison with a set of known masses, the comparison being carried out by means of a balance.

The common balance consists of a light but rigid beam which is balanced at its middle point on a knife edge of agate or steel. At each end of the two arms of the beam is another knife edge from which are suspended two pans in such a manner that they are free to swing in any direction. These two knife edges should be exactly equidistant from the central knife edge on which the beam rests. A long pointer is attached at right angles to the beam and moves over a short graduated scale, the pointer registering zero when the beam is horizontal. A clamping device, worked by a handle on the base board of the balance, is provided to raise the bar from its supports and to clamp it when the balance is not in use. This prevents wear and damage to the knife-edges. The balance is usually enclosed in a dust-proof case, which also protects the instrument from draughts while weighing is in progress. This is essential for a sensitive balance.

For accuracy the arms of the balance must be of exactly the same length and weight, and the weight of the scale pans should also be exactly the same. The balance will then be in equilibrium with the pointer at zero, when resting on its knife edges with no weights in the pans. If now equal weights are put in the two pans they will have equal moments about the central knife edge, and the balance will still come to rest with the arm horizontal and the pointer at zero. Thus if one of the weights is known that of the other can be determined.

EXPERIMENT 9.—To set up a balance and to test its accuracy.

Level the balance by the levelling screws supporting the base until the pillar is vertical. A plumb bob is usually

attached to the pillar for this purpose. Turn the clamping handle very gently, thus setting the beam free. It will probably oscillate about the equilibrium position, or position of rest as it is called, the pointer swinging backwards and forwards over several divisions of the scale. It is not necessary to wait for the pointer to come to rest in order to find the position of rest. Read the turning-point of the pointer on the

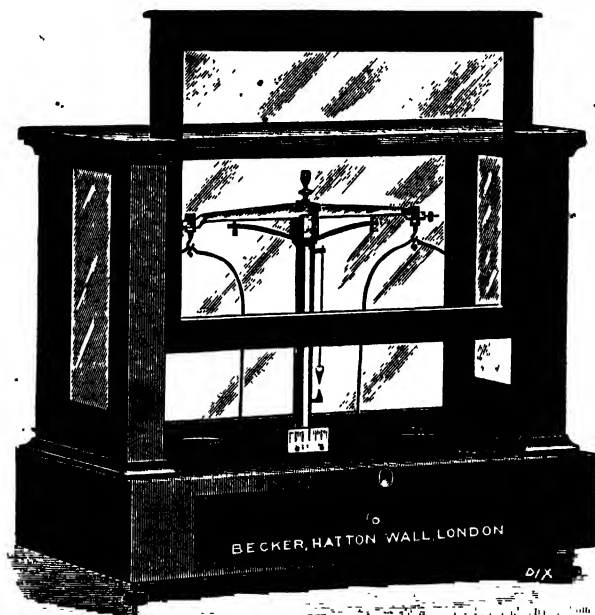


FIG. 8.—An Accurate Balance.

scale at each end of its swing. Take the difference between them and divide it by two. Thus if the pointer moves six divisions to the right and four divisions to the left the position of rest is one division to the right of the zero. The balance should, in fact, always be read in this way, with the beam gently swinging, as it prevents errors due to any tendency which the balance may have to "stick" at any particular point.

If the position of rest for the pointer is not at the zero of the scale, it may be adjusted by fastening a small piece of fine

wire to the lighter pan, and altering the length of wire until a proper balance is obtained. Most accurate balances are furnished with adjusting screws (see Fig. 8) one at each end of the balance arm for producing this adjustment. The screw on the side of the lighter pan is turned so that it moves further from the central knife edge, and thus increases the moment of the arm about its centre. The adjustment of the balance by means of the adjusting screw, however, should not be attempted by the student himself, as inexperienced handling of any part of the beam of the balance is likely to cause damage to the instrument.

If the two arms of the balance are of exactly the same length the balance is now in adjustment. If, however, one arm is slightly longer than the other this is not the case, even if the pointer is at zero when there is no load on the pans. This error can only be rectified by a balance maker. To test whether the balance arms are of the same length proceed as follows. Place a piece of metal (a weight for example) in the left hand pan of the balance and weigh it accurately by finding what weights must be placed in the right hand pan to counter-balance it exactly. Record the weight. Now transfer the object to the right hand pan of the balance, and weigh it again. If the two weights are not exactly the same, the arms of the balance are slightly unequal.

Instead of using very small weights, a rider is often employed. This consists of a piece of bent wire which can rest in any position on the balance beam. The beam is graduated into ten equal parts, the 0 being directly above the supporting knife edge, and the 10 immediately above the support of the balance pan. If the rider is placed on graduation 10, its effect is the same as if it were in the pan itself; but as it is moved nearer the beam its effect becomes smaller (by the principle of moments) so that when resting on graduation 3, for example, its effect on the balance is only 0.3 of its weight. Thus if the rider weighs 1 centigram (0.01) its effect when on graduation 3 of its beam is only 0.3×0.01 or 0.003 gm. The rider can thus be used instead of a set of milligram weights. This is the usual arrangement. A rider weighing exactly 0.1 gm. could, however, be used in a similar way, instead of the centigram weights. This method might be advantageous in an elementary laboratory, where even centigram weights are liable to be lost or mutilated.

RULES FOR THE USE OF A BALANCE

If the following rules are attended to it will be found to facilitate accuracy in weighing, and to reduce risk of injury to a valuable instrument.

1. Keep the balance pans perfectly clean. Powders and substances of any kind likely to leave a mark or stain must never be placed directly on the pan, but must be weighed in some suitable vessel. The outsides of all vessels placed on the pan must be perfectly clean and *dry*.

2. Always move the handle slowly and gently, both in clamping and unclamping, so as to avoid shaking or jarring the beam.

3. On no account must weights be placed upon or taken from the balance pans until the beam has been clamped. Observance of this rule will be facilitated if the student makes a practice of working the handle, and moving the weights with the same hand.

4. Weights must never be touched with the fingers, or held in the hand. Forceps are always provided in the boxes of weights and should always be used.

5. Weights must not be placed anywhere except on the balance pan or in the box.

6. In accurate weighing the final balance is always tested with the balance case closed, to avoid draughts. If the final balance is to be obtained by means of a rider the balance case may be kept closed as soon as the rider comes into use, as the rider can be adjusted from outside the case.

7. The balance case must be left closed when the weighing has been completed, and all weights must be replaced in their proper partition in the box.

If the balance arms are not equal, the correct weight of a body may be found by weighing the body first with the body in the left hand scale pan, and then with the body in the right hand scale pan. It can be shown that if W_1 and W_2 are the weights required to balance the body in the two cases the true weight W of the body is given by

$$W = \sqrt{W_1 W_2}$$

that is to say, the actual weight is the geometric mean of the

two apparent weights. In any reasonable balance W_1 and W_2 will be very nearly the same, in which case we may use the arithmetical mean instead of the geometrical mean without any sensible error. In this case the true weight of the body may be taken as $\frac{1}{2}(W_1 + W_2)$. This method, due to Gauss, is known as the method of double weighing.

EXPERIMENT 10.—To determine the weight of a body (a) by double weighing, (b) by the method of counterpoising. (Borda's method.)

(a) Determine the weight of a given body, say a piece of lead, as described in the previous paragraph, by weighing it first in the right hand pan, and secondly in the left hand pan. Calculate its true weight by taking the geometric mean. Calculate also the arithmetical mean and note the difference, if any.

(b) The body is placed on the right hand scale pan, and is counterpoised by placing lead shot or sand (which should be contained in a small dry beaker) on the other pan, the final adjustment being made with fine sand and scraps of paper. Leaving the counterpoise untouched the body is removed from the right hand pan, and the counterpoise is weighed by placing weights on the right hand pan of the balance. The weight of the body must be exactly equal to the sum of the weights necessary to balance the counterpoise. This method, which is due to Borda, is no more accurate than the method of Gauss, and is distinctly less convenient to work.

Most balances, if they have not been subjected to rough treatment, are wonderfully accurate, and a single weighing is generally sufficient. For very accurate weighing, however, one or other of the two methods just described must be employed. A good physical balance will take a mass of 100 grm. in each pan, and will be perceptibly out of balance when the difference between the weights in the two pans is only 1 mgrm. It is thus capable of detecting a difference in weight of 1 part in 100,000, and weighing is thus one of the physical measurements which can be carried out with the greatest accuracy. In weighing to such a degree of accuracy, however, many factors have to be taken into account, such for example as the effect of the buoyancy of the air on the body weighed and also on the weights used. The magnitude

of this effect depends on the relative densities of the body and the weights. It amounts to as much as 1 part in 1000 in the case of water, but is less for denser substances. The weights themselves would also require to be calibrated, as errors of the order of 1 mgrm. might easily exist in individual weights, especially after they have been in use for some time. Unless these points are taken into account the result will probably not be correct to more than 1 part in 1000, however sensitive the balance may be. This point is often overlooked.

§ 5. MEASUREMENT OF VOLUME

THE volume of a liquid is measured by means of graduated vessels which for scientific purposes are usually made of glass, and for commercial purposes usually of metal (for example, the milkman's pint measure). These contain a specified volume of liquid, at a definite temperature, when filled up to

some mark engraved on the surface of the vessel. The percentage error in estimating when the liquid is exactly level with the mark will be less if the mark is made on a narrow part of the vessel. Measuring flasks are, therefore, made with a narrow neck. Thus if the neck of a litre flask has an area of cross-section of 1 square centimetre and an error of 1 mm. is made in estimating when the liquid is exactly level with the scratch, the error in the volume will be $1 \times \frac{1}{10}$ th cubic centimetres (c.c.). This is an error of only 1 part in 10,000, which is less than other errors incidental to such measurements.



FIG. 9a.—
Measuring
Flask.

A measuring flask (Fig. 9a) only measures a definite fixed volume. For other volumes measuring cylinders



FIG. 9b.—
Measuring
Cylinder.

(Fig. 9b) are used. These are tall narrow cylinders of uniform cross-section. The volume of liquid in such a cylinder is equal to the height to which it rises in the cylinder multiplied by the area of cross-section of the cylinder. The cylinder is usually graduated in cubic centimetres. It is obvious that for accuracy the height should be large compared with the area of cross-section, so that a small error in reading the height may not produce too large an error in estimating the volume. In a laboratory a variety of measuring jars suitable for measuring

different volumes are employed. Photographers, who do not wish to have too many different measures in the dark room, use conical measuring vessels. The narrowest measuring cylinders are usually fitted with a tap at the bottom (Fig. 9, *c*) to allow a measured volume of liquid to be run out into some other vessel. They are then known as burettes.

For very accurate work the volume of a liquid is estimated from its weight, as weight can be determined much more accurately than volume. The weight of unit volume of the liquid, or the density as it is called, is either known, or can be determined by subsequent experiment. (See Experiments 16, 40, 41.)

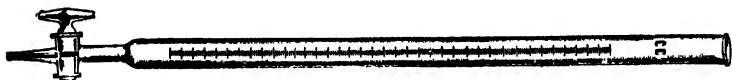


FIG. 9c.—The Burette.

The measurement of the volume of a solid can be made by observing the volume of liquid which it displaces. This may be done in several ways.

EXPERIMENT 11.—To determine the volume of a solid by a measuring cylinder or burette.

Select a measuring cylinder into which the solid will go. The more nearly the solid fits the cylinder the more accurate the measurement will be. About half fill the cylinder with water and read the volume carefully. Care must be taken that the eye is on the same level as the surface of the water. Water in a cylinder has a curved surface, the curvature being more pronounced as the diameter becomes smaller. The lowest part of the meniscus (as it is called) is read, and as this is not in contact with the graduations errors due to *parallax* may arise. Thus if the eye is above the level of the water the reading will be too high; if below the water surface the reading will be too small (Fig. 10, *a*). In the case of a fairly wide cylinder, where the middle part of the water surface will be plane, it is easy to tell whether we are looking horizontally along this plane surface (Fig. 10, *b*). In the case of a narrow tube, such as a burette, the observations will be facilitated by holding a piece of white paper behind the tube.

The solid is now introduced into the cylinder, care being

taken that no water splashes out, and that the solid is completely immersed. The level of the water is again read. The difference between the two readings is equal to the volume of the solid.

Sufficient water must be taken to cover the solid completely, but not enough to rise above the top graduation on the jar. If any air bubbles are entangled on the surface of the solid they must be displaced by gently tapping or shaking the jar. If the solid is lighter than water it must be held below the surface while the reading is being taken, by pushing it down with a thin stiff wire, the volume of which is negligible compared with that of the solid. As an alternative a sinker may

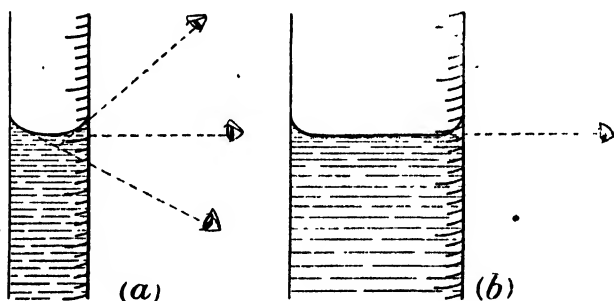


FIG. 10.—Illustrating Errors due to Parallax.

be attached to the solid; the volume of the sinker by itself being determined separately.

If the solid is too large to go conveniently into any measuring cylinder available, the following modification may be adopted :

EXPERIMENT 12.—To find the volume of a solid by a burette.

A beaker is selected just large enough to contain the solid. A mark is made on the side of the beaker near the top, either by a short horizontal file mark, or, if it is desired not to scratch the beaker, by a piece of straight-edged stamp paper. Water is run into the beaker from a burette until its surface is exactly at the level of the mark. The volume of water run in can be ascertained from the readings of the burette. The water is

poured away, and the beaker allowed to drain for half a minute or so. The solid is then placed in the beaker, and the volume of water required to fill the beaker up to the same mark, with the solid in the beaker, is again measured by means of the burette. The difference between the two volumes of water is the volume of the solid.

If the solid is soluble in water some liquid in which it is insoluble must be used instead. The volume of a lump of sugar, for example, could be measured by using paraffin.

The volume of water displaced may also be measured by means of a displacement cylinder.

EXPERIMENT 13.—To measure the volume of a solid by the displacement vessel.

A displacement vessel consists essentially of a vessel with a small aperture near the top. The vessel is filled with water above the level of the aperture and the water allowed to empty itself through the aperture until the flow ceases. The solid is then gently lowered into the vessel and the water which flows out through the aperture is caught in a small weighed beaker, and its weight obtained. Since 1 gm. of water occupies a volume of 1 c.c. the volume of water expelled can be determined. This is obviously the volume of the solid.

A metal can with a short metal tube soldered into the side near the top can be employed. A still better form of displacement vessel is that shown in Fig. 11, which can easily be constructed in the laboratory. A length of quill tubing passes through a cork in the bottom of the vessel (which may consist of an ordinary glass bottle from which the bottom has been cracked off), and reaches to within an inch or so of the top of the vessel. Water will run out through the quill tube until it reaches the level of the upper end. Care must be taken not to shake or jar the vessel during the experiment.

The displacement vessel, though popular in the elementary laboratory, is not of practical importance.

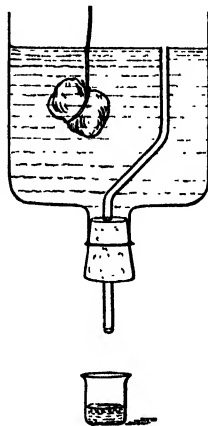


FIG. 11.—A Displacement Vessel.

As in the case of liquids the volume of a solid is obtained most accurately by weighing. (See Experiments 18, 19, 20.)

A measurement which is often required in practical work is to determine the area of cross-section of a capillary glass tube. This is always done in practice by finding the weight of mercury which is required to fill a measured length of the tube. If α is the area of cross-section of the bore of the tube, supposed uniform, and h the length of the tube occupied by the mercury, the volume of mercury is $h \cdot \alpha$, and its mass is therefore $h\alpha\rho$ where ρ is the density of mercury (approximately 13.56 at 15° C.).

EXPERIMENT 14.—To determine the area of cross-section of a capillary glass tube.

The experiment should be carried out as follows. A small porcelain crucible is accurately weighed, and placed on a piece of clean paper in a convenient position on the bench. The glass tube, the ends of which should be cut off square with the length, is carefully dried by warming it gently in a bunsen flame, and at the same time blowing a slow stream of air through it from a pair of foot bellows. One end of the tube is then connected by a short piece of rubber tubing to a small dry funnel. The other end of the glass tube is raised so that it is well above the top of the funnel, and dry mercury is poured into the funnel. It is usually possible to hold the funnel and tube in one hand, leaving the other free to manipulate the mercury bottle. If not, assistance should be obtained. The upper end of the glass tube is now gently lowered, and the mercury will be found to run along it in a continuous column which will finally reach the open end. As soon as the mercury begins to run out (the operation should be performed over a wooden mercury tray to avoid loss of mercury) a finger is placed over the end to close it. The funnel end is then raised well above the level of the end which is now stopped by the finger, and the funnel and rubber tube are removed. The glass tube is now completely full of mercury. Place a second finger over the upper end. Hold the lower end over the weighed beaker and take the lower finger away. The mercury can then be allowed to flow slowly, and without splashing into the weighed crucible. The weight of mercury is found by again weighing the crucible

MEASUREMENT OF VOLUME

and its contents. The length of tube is found by placing it on a metre scale. The area of cross-section can then be calculated as in the following example:

Weight of crucible empty. 8.265 gm.

 " " and mercury 15.353 "

Weight of mercury = 7.088 "

Length of tube = 52.35 cm.

Temperature = 15° C.

Density of mercury at 15° C.

= 13.56 gm. per c.c.

$7.088 = 52.35 \times \alpha \times 13.56$

$\alpha = \frac{7.088}{52.35 \times 13.56} = 0.0100 \text{ sq. cm.}$

This previous experiment measures the *average* area of cross-section of the tube. The uniformity of the tube can be tested by introducing a short thread of mercury (about 3 cm. long) into the tube and carefully measuring its length when, say, near one end of the tube. The thread is then displaced by gently tilting the tube until it occupies the next section of the tube, when its length is again measured. If the tube* is of uniform cross-section the column will be of exactly the same length in all positions. If the column varies in length the tube is not uniform. This process is known as *calibrating* the tube.

§ 6. MEASUREMENT OF DENSITY AND SPECIFIC GRAVITY

THE specific gravity of a solid or liquid is the ratio of the mass of a given volume of the substance to the mass of an equal volume of water (at its maximum density).

The density of a substance is the mass of unit volume of it.

The former is a ratio, and is independent of the units employed in measuring it. The latter depends both on the unit of mass and the unit of volume. In constructing the centimetre-gram-second system, the unit of mass, the gramme, was defined as being the mass of 1 cubic centimetre of water at its maximum density (*i.e.* at a temperature of 4°C.). Thus the density of water at 4°C. on the C.G.S. system of units is 1 *by definition*. The density of water does not vary much with change of temperature. Taking the density as unity at 4°C. at 15°C. it is 0.99913 and at 30°C. it is 0.99567. Thus at ordinary laboratory temperatures the volume of a quantity of water in cubic centimetres is equal to its weight in grams to an accuracy of at least 1 in 200. This is a very convenient arrangement. It also follows that the density of a substance in grams per c.c. is numerically equal to its specific gravity. The density of a substance measured in lbs. per cubic foot is equal to its specific gravity multiplied by the weight of one cubic foot of water, which is approximately 62.3 lb. In nearly all cases it is the specific gravity of the substance which is experimentally determined. The density of a solid may, however, be determined directly if it has some definite geometrical shape, so that its volume may be calculated from its linear dimensions.

EXPERIMENT 15.—To determine the density of a geometrical solid.

A cylindrical brass rod about 6 cm. long and not less than 2 mm. in diameter may be used. Measure the length h of the rod by vernier calipers (or by scale if it is too long for the

calipers), and its diameter d by means of a screw gauge. Hence determine its volume, $V (= \frac{1}{2}\pi d^2 h)$. Weigh the body on an accurate balance and let W be the weight. The density

of the substance is $\frac{W}{V}$.

In most cases, however, the substance cannot easily be obtained in an accurate geometrical shape. It is then more convenient to determine its specific gravity. This can be done most accurately and conveniently by means of a specific-gravity bottle.

THE SPECIFIC GRAVITY BOTTLE

The specific gravity bottle, or density bottle (Fig. 12) consists of a small glass flask with a slightly conical neck into which fits a slightly conical glass stopper. The stopper and neck are ground together so that the stopper always sinks into exactly the same position in the neck. A fine hole is drilled through the stopper to allow excess fluid to escape. If the bottle is filled with liquid and the stopper inserted, it sinks into place in the neck, any excess of fluid being forced out through the hole. The bottle when filled thus always contains exactly the same volume of liquid (at a given temperature).

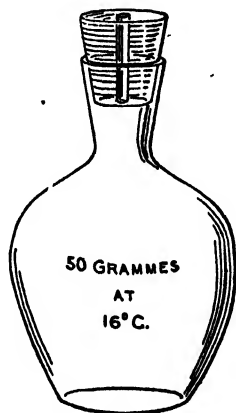


FIG. 12.—The Specific Gravity Bottle.

EXPERIMENT 16.—To determine the specific gravity of a liquid by the specific gravity bottle.

The bottle is washed and dried. The safe way of drying a bottle is first to allow all excess water to drain from it. Then place inside the bottle the end of a fairly long piece of glass tubing the other end of which is attached to a pair of foot bellows. Heat the portion of the glass tube outside the bottle by a bunsen flame, at the same time sending a current of air through it from the bellows. The warm air very quickly evaporates the moisture remaining in the bottle.

Let W_0 be the weight of the empty dry bottle, W_1 the weight of the bottle filled with water, and W_2 the weight of the bottle filled with liquid. Then the specific gravity of the liquid is obviously

$$\frac{W_2 - W_0}{W_1 - W_0}$$

$$\text{Specific gravity of salt solution} = \frac{72.28 - 12.24}{62.34 - 12.24} = \frac{60.04}{50.10} = 1.195$$

Note.—The volume of the bottle is sometimes engraved upon it by the maker. This value should be regarded as approximate only, and should in no case be employed in the calculations. The weight of water filling the bottle should always be found by direct experiment as in the example.

Dry the outside of the bottle and fill it completely with water with the same precautions as in the previous experiment,

MEASUREMENT OF DENSITY

and weigh. Break the solid into fragments sufficiently small to pass through the neck of the bottle, and weigh out sufficient of the fragments to about half fill the bottle. Pour some of the water out of the bottle and insert the fragments, shaking well to dislodge all air bubbles. If the solid is of such a nature as to entangle a large quantity of air (in the case of granulated metal, for example) it may be necessary to boil the water and solid in the bottle in order to get rid of all the air. After boiling, the bottle must be allowed to cool down to air temperature before proceeding. Having dislodged all the bubbles the bottle is again completely filled up with water, as before, and the weight of the bottle which now contains the solid and water is found. Let W_1 be the weight of the bottle filled with water, W_2 the weight of the solid placed in the bottle, and W_3 the weight of the bottle containing the solid and water. Then $(W_1 + W_2 - W_3)$ is the weight of water expelled from the bottle by the solid; that is to say, the weight of a volume of water equal to the volume of the solid. Hence

$$\text{Specific gravity of solid} = \frac{W_2}{W_1 + W_2 - W_3}$$

Note that the weight of the dry bottle is not required in this determination. There is no need, therefore, to dry the bottle before commencing the experiment. It should, of course, be washed out with water before filling.

If the solid is soluble in water some liquid in which it is insoluble must be used. In this case $(W_1 + W_2 - W_3)$ is the weight of a volume of this liquid equal to the volume of the solid. The volume of this liquid is its weight divided by its density. Hence, if the density of the liquid is known, that of the solid can be calculated.

The specific gravity bottle may be used to find the VOLUME of any solid sufficiently small to be placed within it.

EXPERIMENT 18.—To determine the average volume of one of the given screws.

The bottle is filled with water as in the previous experiment, and placed on the balance pan, together with a known number, n , of the screws, and the whole is then weighed. The screws are then inserted into the bottle, as in the previous

experiment, the bottle again filled with water and weighed. The difference between these two weighings is the weight of water expelled by the screws. Since one gram of water occupies 1 cubic centimetre, the volume of the screws in cubic centimetres is equal to the loss in weight in grams. Dividing this by the total number of screws placed in the bottle, the average volume of a single screw can be determined.

The same method can be employed for determining the volume of a length of wire. If the length of the wire is measured before it is twisted up and inserted in the bottle, the mean diameter of the wire can be determined, since the volume is equal to $\frac{1}{4}\pi d^2 l$. Unless the wire is thick, or unless a considerable length of it is used, the amount of water expelled will be too small to be determined with much accuracy.

ARCHIMEDES' PRINCIPLE

In some cases it may be undesirable to reduce the solid whose specific gravity is required to fragments. This was the problem which faced Archimedes when he had to determine the specific gravity of the crown of Syracuse. In such a case the specific gravity can be determined by means of ARCHIMEDES' PRINCIPLE, that *the difference between the weight of an object in air and its weight when completely immersed in a liquid is equal to the weight of the liquid displaced by the object—that is, to the weight of a volume of that liquid equal to the volume of the object.*

EXPERIMENT 19.—To determine the specific gravity of a solid by Archimedes' principle.

In order to weigh a solid when completely immersed in a liquid, balances, known as hydrostatic balances (Fig. 13), are made in which one of the ordinary scale pans can be replaced by a pan so much shorter than the other that a beaker of liquid may be placed between this scale pan and the base of the balance. A hook is provided beneath the scale pan from which the solid may be suspended. The short scale pan is, of course, weighted so that its mass is exactly the same as that of the other pan. Failing a hydrostatic balance, a wooden bridge can be made which stands astride the scale pan, without touching it. The beaker of liquid can be placed on this bridge, and the solid suspended from a hook at the top of the scale pan, just below the balance beam.

To perform the experiment the solid is suspended from the hook by means of a very fine wire, or horse hair, and weighed in the usual way. Ordinary cotton thread may be used, but is not so suitable as it absorbs moisture when hanging in the liquid, and its effective weight is, therefore, not constant. The bridge is placed over the pan, care being taken that it does not interfere in any way with the motion of the pan, and a beaker of water placed on the bridge, so that the solid is completely immersed.

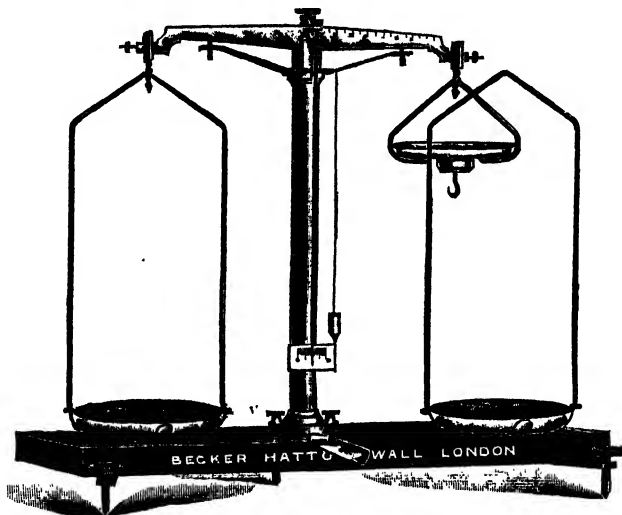


FIG. 13.—The Hydrostatic Balance.

If the solid does not sink in the water the procedure will have to be modified as explained later. If any air bubbles are visible on the sides of the solid these must be brushed off, as they would tend to buoy up the solid. The formation of air bubbles can be largely prevented by using water which has been recently boiled for some time and then allowed to cool to room temperature. The solid is now weighed completely immersed in liquid. Let W_1 be the weight of the solid in air, and W_2 its weight in water. Then $W_1 - W_2$ is the weight of a volume of water equal to the volume of the solid, and the specific gravity of the solid is, therefore,

$$\frac{W_1}{W_1 - W_2}$$

The weight of the wire by which the solid is suspended has been neglected. If necessary, it can be taken into account by weighing it separately and subtracting the weight of the wire from that of the wire and solid. The weight of the wire will not affect the denominator, as it comes into both the weights, while the weight of water displaced by the portion of the wire immersed will certainly be negligible. The effect of the buoyancy of the air on the solid when weighed in air, and of temperature on the density of water, have also been neglected. Corrections for these can easily be made, but are outside the scope of our present work. They will generally be less than 1 part in 500.

This experiment affords the most accurate method of finding the volume of any solid which is too large to put in a specific gravity bottle.

If the solid floats, that is, if its density is less than that of water, we may proceed as follows :

EXPERIMENT 20.—To find the density of a solid which is lighter than water.

Weigh the solid. Fasten to it a sinker (a bob of lead, for example) sufficiently heavy to sink the solid, so that the sinker hangs well below the solid itself, and suspend them by a fine wire from the hook on the balance pan. Place the beaker of water on the bridge so that the sinker is completely immersed, while the body is completely above the surface of the water, and weigh. Now adjust the supporting wire so that both the sinker and the body are completely immersed in water, and weigh again. The difference between these two weighings is obviously the weight of water displaced by the body itself, since the sinker was immersed during both weighings. Calculate the specific gravity of the solid as in the following record of a determination of the specific gravity of paraffin wax :

Weight of wax in air = 54.51 gm.

Weight of wax in air and sinker in water = 295.69 "

" and sinker both immersed = 144.95 "

Weight of water displaced by wax = 60.74 "

\therefore Specific gravity of wax = $\frac{54.51}{60.74} = 0.898$

MEASUREMENT OF DENSITY

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The specific gravity of a liquid can be determined by Archimedes' principle, using a sinker whose weight is known in air and in the liquid and the liquid to be tested.

EXPERIMENT 21.—To determine the specific gravity of a liquid by Archimedes' principle.

The sinker may very conveniently be a large glass stopper. Glass is sufficiently dense to sink in most liquids, but not so dense that a reasonable volume of it will be too heavy for the balance, as might be the case if a heavy metal were chosen. It is also not likely to be chemically acted upon by any of the liquids used. The sinker is suspended in the usual way, and weighed first in air ($=W_0$), then completely immersed in water ($=W_1$), and finally, after being dried with a duster, is weighed when completely immersed in the liquid whose specific gravity is required ($=W_2$). $W_0 - W_1$ is the weight of water displaced by the sinker, while $W_0 - W_2$ is the weight of the liquid displaced by the sinker. In each case the sinker displaces its own volume of fluid.

Hence

$$\text{Specific gravity of the liquid} = \frac{W_0 - W_2}{W_0 - W_1}$$

EXPERIMENT 22.—To determine the specific gravity of a liquid by the U-tube.

To compare the specific gravities of two liquids which do not mix, a glass U-tube with arms about 30 to 50 cm. in length may be used. The tube is clamped in a vertical position and a quantity of the denser liquid is poured in so that it occupies about 10 cm. in each limb of the tube. The second liquid is then poured gently down one limb until the two liquids occupy suitable lengths of the tube. The heavier liquid must still occupy the bend of the tube. If necessary, more of this liquid may be poured down the other limb. The position of affairs is shown in Fig. 14, *a*, where *D* is the surface separating the two liquids and *B* and *A* the levels of the free surfaces of the lighter and denser liquids. By means of a metre scale held vertically measure the vertical heights of *A*, *B*, and *D* above the level of the table. $B - D$ and $A - D$ will then be the heights of the two columns of liquid above the surface of separation. The densities of the two liquids are inversely proportional to their heights above this surface.

If desired, the U-tube may be permanently mounted on a stand, with a scale attached. The different heights can then be read off on the scale.

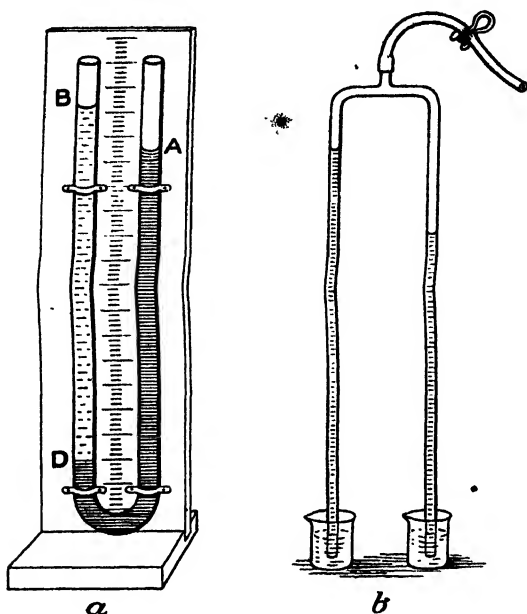


FIG. 14.—Determination of the Relative Densities of Two Liquids by Balancing Columns. (a) U-tube Method; (b) Hare's Apparatus.

Example :

Comparison of the Specific Gravities of Oil and Water

Height of common surface above table = 6.75 cm.

" oil " " = 48.2 "

" water " " = 32.0 "

$$\text{Specific gravity of oil} = \frac{32.0 - 6.75}{48.2 - 6.75} = 0.80$$

This method can obviously only be employed with pairs of liquids which do not mix. It can, however, be modified for use with any pair of liquids, as follows :

EXPERIMENT 23.—To determine the relative densities of liquids by Hare's apparatus.

In this modification a short side tube is sealed into the bend of the U-tube, a length of rubber tubing with a stop cock

being attached to the side tube (Fig. 14, δ). The U-tube is then clamped in a vertical position, with the open ends of the two limbs dipping into separate beakers containing the two liquids whose densities are to be compared. The pinch cock is opened and air is sucked out through the rubber tube, until the lighter of the two liquids has risen nearly to the top of the corresponding limb. The pinch cock is then closed, and the heights of the two columns of liquid above the level of the liquid in the corresponding beaker is observed. If the liquids are non-corrosive this measurement can be made most conveniently by placing the metre scale vertically alongside the tube, with its lower end resting on the bottom of the beaker, and reading off the two levels directly on the scale. The densities of the two liquids are inversely proportional to the heights of the two columns. If one of the liquids is water, the measurements will give the specific gravity of the other liquid.

Example :

Determination of Specific Gravity of Salt Solution

Level of salt solution in limb	= 35.2 cm.
" " beaker	= 3.5 "
∴ Height of column of salt solution	= 31.7 "
Level of water in limb	= 41.6 "
" " beaker	= 4.1 "
∴ Height of water column	= 37.5 "
Specific gravity of salt solution	= $\frac{37.5}{31.7} = 1.18$

§ 7. MEASUREMENT OF TIME

THE SIMPLE PENDULUM

TIME is generally measured by means of clocks or watches, which, as the intervals of time which we wish to measure in the laboratory are often quite short, should be fitted with a centre seconds hand, so that fractions of a second can be read. Stop clocks and stop watches, that is to say clocks or watches which can be started and stopped by pressing a lever, are convenient as they leave the observer free to watch closely for the occurrence of the events which it is required to time. They are not usually quite so accurate as the non-stop variety, as the delicate mechanism of a watch is not improved by repeated starting and stopping. A stop watch, which has seen much service in an elementary laboratory, cannot be implicitly relied on. In experiments where the absolute value of the interval of time is required, and not merely the ratio of two intervals, the stop watch should always be rated by comparison with a standard clock. In any case the stop watch should be examined before use to see (a) that the seconds hand begins to move as soon as the knob or lever is pressed, and (b) that on pressing the lever or knob to stop the watch the seconds hand stops immediately in the position it had reached without jerking forwards or backwards, a defect which develops in some types of watches as the mechanism begins to wear. The following experiments on the simple pendulum will serve as practice in the art of timing:

EXPERIMENT 24.—To find the time of vibration of a simple pendulum.

Theoretically a simple pendulum consists of a heavy particle hanging at the end of a weightless inextensible string. If such a pendulum is displaced and then let go, it will oscillate backwards and forwards about its vertical position, each complete oscillation occupying exactly the same time if the oscillations are not too large. In practice we can approximate sufficiently

closely to the ideal simple pendulum by hanging a small lead sphere about one centimetre in diameter at the end of a fine thread (a plaited cord such as is used for fishing line is more satisfactory than ordinary thread). The length of the pendulum is then the distance from the point of support to the centre of the bob. The time taken by the bob in swinging to and fro is called the time of a complete vibration, or the period of the pendulum. It is the time occupied by the pendulum in swinging from one extreme of its path to the other and back again. It can, however, be observed more accurately by taking the interval between the moment when the string is passing through the vertical position, say from left to right, and the moment when it next passes through the same position *in the same direction*. The reason for this is that the pendulum is moving most rapidly in this position and hence the exact instant when it is passing through this position is more easy to observe.



The string of the pendulum may be conveniently supported by threading it through a small brass eye, screwed into the clamp of a retort stand. The free end of the string may be attached to the stand itself, or may be wound on a small reel, which turns stiffly on its support, as illustrated in Fig. 15. The length of the pendulum can thus be readily altered.



FIG. 15.—A Simple Pendulum.

Let the pendulum come to rest with the string vertical, and place a short vertical ruler immediately behind it to serve as a mark. Displace the bob of the pendulum to one side and let it go, so that it swings backwards and forwards with the string always in the same plane. As the string passes the ruler from left to right start the stop watch and begin to count the swings, starting at *nought*. Count *one* when the string next passes the ruler from left to right, two at the next transit in the same direction, and so on up to fifty. Stop the watch exactly at the

fiftieth transit, and record the time taken. This will be the time of fifty complete vibrations.

If a stop watch is not available a clock with a centre seconds hand can be used, but two observers will be required. The first observer is responsible for recording the times, the second watches the pendulum and does the counting. The second observer sets the pendulum swinging, and when ready to begin counting warns his colleague by saying "ready," and then, as the pendulum passes the mark from left to right, he gives the signal by tapping smartly on the bench with a pencil, and begins his count, the first transit from left to right after the signal being counted *one*. As he approaches the end of his count he again warns his colleague, and at the instant the pendulum is making the last transit to be counted, he again signals by a second tap of the pencil. The first observer, with his eyes on the clock, observes and records the exact times at which he hears the two taps.

Both methods require practice. It is much more possible to make errors in counting up to fifty than the student would imagine, until he has tried it. A very common error is to count one at the first transit instead of nought. Practice in starting and stopping the stop watch, or in reading the exact time of a signal on the clock, is also required. The observations should be repeated without varying the pendulum in any way, until consistent resultants can be obtained.

The swings of the pendulum are only isochronous if the arc through which it is swinging is small. As, however, the amplitude of the vibrations gradually dies down, owing to the friction of the air and other causes, the initial amplitude must not be too small, or the last vibrations may not be sufficiently large to be observed with accuracy. If the bob is drawn aside, so that the string makes an angle of about 10° with the vertical, these conditions will be fulfilled with sufficient accuracy, unless the bob is unduly light.

EXPERIMENT 25.—To determine how the time of vibration of the simple pendulum varies with the length of the pendulum.

Adjust the pendulum so that its length is about 25 cm. Measure the length accurately by measuring the distance from the point of support to the top of the bob with a metre scale, and adding on one-half of the diameter of the bob. (The

bob is supposed to be spherical.) Time the pendulum as in the previous experiment.

Increase the length of the pendulum by about 25 cm. or so, and repeat the observations. Continue in this way until you have reached the full length of the string.

It can be shown mathematically that the square of the time of vibration is directly proportional to the length of a simple pendulum. Thus if t is the time of one complete vibration, and l the corresponding length of the pendulum, t is proportional to t^2 , or $\frac{l}{t^2}$ should be constant. The results may be recorded as in the following example :

Diameter of bob = 1.60 cm. \therefore Radius of bob = 0.80 cm.				
Distance from Point of Support to Top of Bob.	Length of Pendulum = l .	Time of 50 Swings.	t .	$\frac{l}{t^2}$
25.15 cm.	25.95 cm.	51.0 sec.	1.020	24.9
50.60 "	51.40 "	72.4 "	1.448	24.6
78.25 "	79.05 "	90.0 "	1.800	24.4
105.70 "	106.50 "	103.6 "	2.072	24.9
151.35 "	152.15 "	124.2 "	2.484	24.6
198.20 "	199.0 "	141.2 "	2.824	24.9
Mean . . .				24.7

The quotients in the last column are constant, within the limits of experimental error. Hence the square of the period is proportional to the length of the pendulum.

EXPERIMENT 26.—To determine the value of g (the acceleration due to gravity) by the simple pendulum.

It can be shown that the time of swing of a simple pendulum is given by

$$t = 2\pi \sqrt{\frac{l}{g}}$$

where g is the acceleration due to gravity, that is the acceleration experienced by a freely falling body. Hence

$$g = 4\pi^2 \frac{l}{t^2}$$

Determine the mean value of $\frac{l}{T^2}$ as in the previous experiment. If, however, we require only to calculate the value of g it will be advisable to make our observations only with the greater lengths of the pendulum, as both the length and the time can be measured more accurately when they are large. It will also be better to confine ourselves to two or three different lengths, timing each length three times to ensure greater accuracy and to eliminate possible errors in counting, etc. From these observations calculate the mean value of $\frac{l}{T^2}$. Taking the previous values we have

$$g = 4\pi^2 \frac{l}{T^2} = 4\pi^2 \times 24.7 \\ = 976 \text{ cm. per sec. per sec.}$$

If the experiments are repeated using bobs of different mass and different substances, *e.g.* brass, iron, etc., it can be shown that the value of g is independent of the nature and the mass of the bob. Hence all bodies fall with the same acceleration—a very important result.

§ 8. THE USE OF GRAPHS

THE result of a series of observations can be represented graphically by means of squared paper such as was used in Experiment 7. Let us take the results of the pendulum experiment as an example. Suppose we plot the values of l

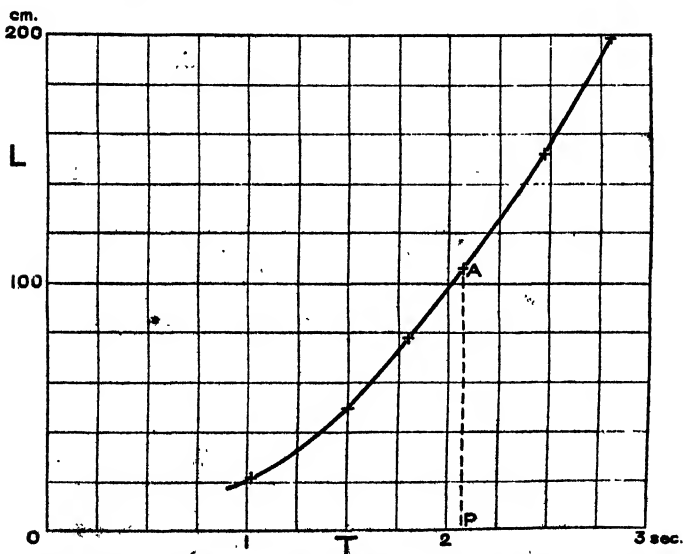


FIG. 16.—Graph connecting Length L , and Time T , for a Simple Pendulum.

along the vertical axis, taking, say, one small square to represent 2 cm. and the corresponding times of swing along the horizontal axis of the paper, taking, say, one small division to represent $\frac{1}{10}$ th of a second, we can represent our observations by a series of points, the vertical distances of which represent the pendulum lengths, while the horizontal distances represent the corresponding times. If we draw a smooth curve to run as nearly as possible through all these points, we obtain a graph

which represents the relation between the length of a simple pendulum and its time of swing (Fig. 16).

A graph is always graduated, and read, not in "squares," but in the quantities which they represent. Thus in the example given (Fig. 16) the large squares would be numbered 20 cm., 40 cm., etc., along the vertical axis, and 0.25 seconds, 0.5 seconds, and so on, along the horizontal axis quite irrespective of what their actual lengths may be. (For the sake of clearness in printing the small squares are not shown in the figure.) All "lengths," "areas," and so forth on a graph are measured in these arbitrary units. Thus the area of one large square on the graph we are considering would not be one square inch, or one square cm., but 10 cm. \times 0.25 seconds, that is 2.5 cm. seconds. Similarly the

ratio $\frac{AP}{OP}$ which we may call the "tangent" of the "angle

AOP," must be read as $\frac{106.5}{2.072}$ irrespective of what the actual

lengths AP and OP may be in centimetres or inches.

If the graph has been accurately drawn we can obtain much information from it. Suppose, for example, we wish to know the length of the pendulum which would make one complete vibration in exactly 2 seconds. We run a pencil up the vertical line numbered 2 seconds on the horizontal scale until it meets the graph. The corresponding reading of this point of the graph on the vertical scale is 99.4 cm., which gives at once the required length. Conversely the time of swing of a pendulum of given length can be determined from the graph.

The graph between l and t curves rapidly upwards, showing that the length of the pendulum varies more rapidly than the corresponding time of swing. The curve is, in fact, a portion of a parabola. It is, however, far from easy to identify by mere inspection the exact form of a curve. The only kind of line which we can identify immediately is a straight line, so that if *if possible a series of results should always be plotted in such a way that the resultant graph is a straight line.* In the case of the simple pendulum we can obtain a straight line by plotting l against t^2 instead of against the first power of t (Fig. 17). The fact that on plotting l against t^2 the resulting graph is a straight line, shows that l is directly proportional to t^2 .

On plotting a set of actual observations, such as those on the pendulum, it will probably be found that it is impossible to draw any straight line which will pass accurately through all the points. This is due to unavoidable slight errors in experiment, affecting the individual points. The graph must be

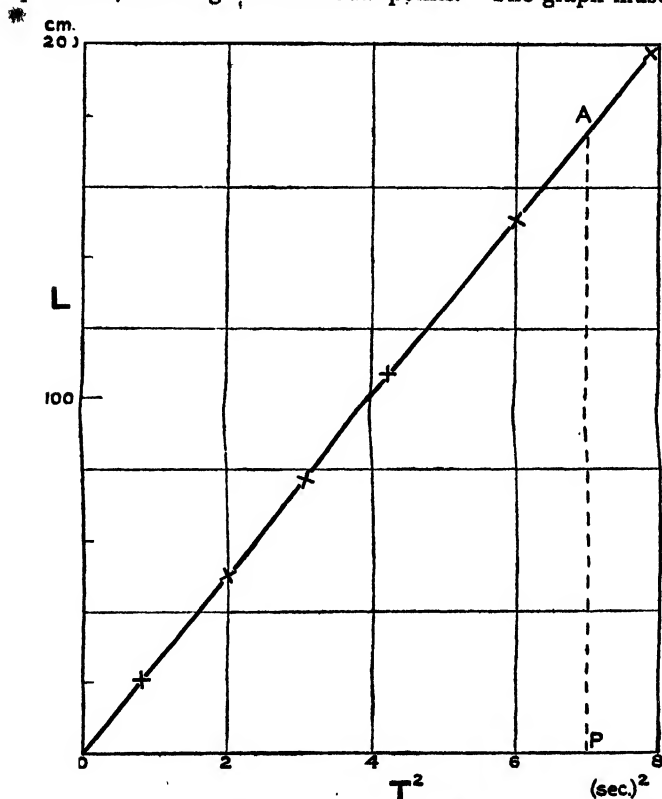


FIG. 17.—Graph connecting Length, L , with T^2 , for a Simple Pendulum.

drawn so as to lie as evenly as possible among the points. An ordinary ruler is not suitable for this purpose, as it conceals some of the points when it is placed on the graph paper. A transparent celluloid scale is most convenient. Failing this a piece of thin black thread may be taken and stretched across the paper and adjusted so as to lie as evenly as possible among

the experimental points. Two points on the stretched thread can then be marked on the paper by pin pricks, and a straight line ruled through the pin pricks with a ruler.

If the graph is not a straight line a smooth curve must be drawn to lie as evenly as possible among the points. Flexible rules with devices for clamping can be bought for this purpose which can be adjusted to the points. A thin flexible steel strip answers the purpose sufficiently well. This is bent to fit the points, the different parts of the strip being kept in position by the fingers of both hands. When satisfactorily adjusted a line may be drawn along it with a fine pencil by a fellow-student.

It is assumed that the graph of a set of observations will be a smooth curve. Merely to join up the individual points either by a number of straight lines or by a "wiggly" curve (as is often done) is bad physics. If the individual points are so bad that it is impossible to find a smooth curve to fit them, they should be repeated.

If the graph reduces to a straight line, the mean result of all the observations can be calculated from the graph. Fig. 17 is the graph obtained by plotting l against T^2 for the simple pendulum used in Experiment 25. If A is any point on the graph, OP represents the value of T^2 for a pendulum the length of AP. The ratio $\frac{AP}{OP}$ is the same for all points on

the graph, and is obviously the mean value of $\frac{l}{T^2}$ as determined

by the observations. To determine the value as accurately as possible the point A will, of course, be taken somewhere near the end of the graph as shown in the figure, as the percentage error in reading the graph becomes less as the distance to be read becomes greater.

One advantage of representing results on a graph is that it enables us to detect and allow for experiments in which the errors of observation happen to be unusually large. Such observations are almost sure to occur in a series of experiments. The points corresponding to such observations will be more or less obviously "off" the graph, and the student will automatically assign less importance to them when deciding where to draw his line. The mean deduced from a graph will thus be more reliable than the arithmetical mean of the whole series

of observations. The method is not absolutely perfect, as it leaves something to the judgment, and, possibly, the bias of the experimenter. *More accurate methods of allowing for the errors in individual observations have been devised, but these are too intricate to be dealt with in this volume.*

ADDITIONAL EXERCISES AND EXAMINATION QUESTIONS.—I

1. Determine the average thickness of the given bronze coin (a penny). The density of bronze is 8.80 gm. per c.c. (Determine the volume from the mass and the density, and the area of the face from the diameter.)

2. Determine the volume of the given cylinder directly. Determine its length and diameter and deduce the value of π . (The volume is most accurately determined by the method of Experiment 19.)

3. Determine the area of cross-section of the bore of the given glass tube. Measure its external diameter, and deduce the actual volume of glass in one centimetre length of the tube.

4. Make a solution of common salt in which a drop of aniline will just remain suspended in any position. Determine the density of the salt solution, and deduce that of aniline.

5. Wrap copper wire round the given piece of paraffin wax until it just sinks in water. Determine the weight of the wax and the wire, and deduce the specific gravity of the wax. The specific gravity of copper is 8.93.

6. Find the time of vibration of the two given simple pendulums, and deduce the ratio of their lengths.

7. Float the given lead pencil in a vertical position in water. Determine the volume immersed and deduce the weight of the pencil.

BOOK II

MECHANICS AND HYDROSTATICS

§ 9. VELOCITY AND ACCELERATION

THE velocity and acceleration of a body are generally measured in the laboratory by noting the times at which it passes each of a series of points along its path. A body which covers equal lengths of path in equal intervals of time (no matter how small) is said to move with uniform velocity. If the velocity is uniform, it is measured by dividing the space passed over by the time taken to describe it. Uniform velocity is, however, very difficult to realise in practice, and the only instance of uniform velocity which can be easily produced in the laboratory is the case of a very small body, such as a small glass bead, falling through a viscous liquid such as thick oil or glycerine.

In most cases the velocity varies, that is, the body has an acceleration. The velocity at any point can best be obtained by constructing a space-time diagram.

EXPERIMENT 27.—To investigate the motion of a fly-wheel rolling down an inclined pair of rails.

A fly-wheel, consisting of a circular brass disk, about 12 cm. in diameter and 2 or 3 mm. thick, with an axle consisting of a brass rod about 3 mm. in diameter, is convenient. A suitable track can easily be made of a pair of straight-edged boards about 1 metre in length and 5 cm. in depth, fastened parallel to each other, but about 3 cm. apart, so that the fly-wheel can rotate between them (Fig. 18). The experiment can also be performed with a marble rolling down a straight groove, but the motion of the marble is much more rapid than that of the fly-wheel and is, therefore, less easy to time accurately.

Make a pencil mark near one end of the rails, so that when the axle is exactly over the mark the wheel is not quite touching the top of the track. Starting from this mark, mark off a series of lines exactly 10 cm. apart. Raise the zero end of the track on a small wooden block and place a heavy weight against the lower end to prevent it slipping. The track should make an angle of from 5° to 10° with the bench. Place the fly-wheel on the track, with its axis on the zero mark and at right angles to the length of the track, holding it in position by placing one finger lightly on the top of the rim. Now, simultaneously lift this finger, thus allowing the wheel to begin to rotate, and start a stop watch. When the axle is directly over the 10 cm. mark, stop the watch. This gives the time taken by the wheel to cover the first 10 cm. of its path.

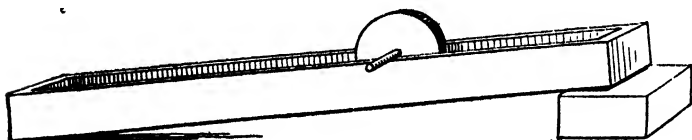


FIG. 18.—Motion of a Fly-wheel on Inclined Rails.

Bring the wheel back again to its starting-point, and repeat the observations, this time, however, allowing the wheel to go to the 20 cm. mark. Determine in this way the time taken by the wheel to cover 10, 20, 30, etc. cm. of its path. The vertical heights of two of the marks on the rails above the bench should also be recorded, as this will enable us to calculate the slope of the track.

It can be shown that the fly-wheel rolls down the plane with a constant acceleration. Thus, if s is the space passed over from the starting-point, t the time taken, and a the acceleration,

$$s = \frac{1}{2}at^2 \text{ or } \frac{s}{t^2} = \frac{1}{2}a.$$

The test for motion in a straight line with constant acceleration is, therefore, the constancy of the ratio $\frac{s}{t^2}$. To test this we may either work out the ratio for each of our experimental observations, or, better still, we may plot s against t^2 and note how far the experimental points lie on a straight line. Both methods

should be employed. The results may be recorded as in the following example :

Height of zero mark on rails above bench = 13.6 cm.			
,, 60 cm. ,, ,, = 7.5 cm.			
$\therefore \sin \theta = \frac{6.1}{60} = 0.1017 \quad \theta = 5^\circ 50'.$			
$s.$	$t.$	$s^2.$	$\frac{s}{t^2}$
10 cm.	5.0 sec.	25.0	0.40
20 ,,	7.2 ,,	51.8	0.39
30 ,,	8.6 ,,	74.0	0.41
40 ,,	10.2 ,,	104.0	0.39
50 ,,	11.4 ,,	130.0	0.38
60 ,,	12.4 ,,	154.0	0.39
Mean . .			0.39
Acceleration is constant and equal to $2 \frac{s}{t^2} = 2 \times 0.39 = 0.78 \text{ cm./sec./sec.}$			

This experiment shows that unless the acceleration is very small we cannot time the body with any great accuracy by means of a stop watch over the comparatively small distances which we can work with in the laboratory. It is generally more convenient to find the distance covered in equal intervals of time than to find the times taken to cover equal distances. The following experiment illustrates a method often employed :

EXPERIMENT 28.—To find the acceleration of a trolley on an inclined plane.

The trolley consists of a piece of board about 30 cm. long and 6 cm. wide, fitted with four easy-running wheels. Ball-bearing pulleys make excellent wheels. It will be an advantage if the trolley is weighted with lead to increase its mass. A wooden plank about $1\frac{1}{2}$ metres in length carefully planed will serve as a track. For convenience this may be hinged at one end to a similar board which serves as a base. It

can then be inclined at any angle to the horizontal by inserting wooden blocks between the track and the base (Fig. 19).

For timing purposes, a long flexible steel strip is rigidly clamped at one end of the track with its free end projecting about half way up the track. The free end carries a small brush at such a position that it just touches the surface of the trolley when the latter is resting on the track. If the end of the strip is drawn aside and let go it will oscillate backwards and forwards, each complete oscillation being made in exactly the same time. The time of oscillation varies with the length of the free part of the strip, and can thus be adjusted.



FIG. 20.—
Curve Trace
for Motion
of a Trolley
on an In-
clined Plane.

To perform the experiment a strip of white paper is fastened to the surface of the trolley with drawing-pins, and the brush is dipped in ink. The track is adjusted at some suitable angle, say 10° or 12° to the horizontal, and the trolley is placed upon it so that the lower end of the paper is just in contact with the brush. The free end of the steel strip is displaced sideways to the edge of the paper and let go, and simultaneously the trolley is also released and begins to run down the plane. As the trolley moves through increasing distances in equal intervals of time the brush will trace a wavy line on the paper, the length of successive waves increasing rapidly as the motion proceeds. A curve is thus obtained as illustrated in Fig. 20. Before removing the paper from the trolley, bring vibrating strip to rest, replace the trolley in its original position, and allow it to run down the plane again. The brush will then mark a

straight line down the length of the paper intersecting each of the curves.

Now the portion of the curve ABC represents one complete vibration of the strip, and similarly CDE, EFG, etc., also represent complete vibrations. Hence AC, CE, EG, ... are the distances moved by the point in one, two, three, ... vibrations of the strip. If the acceleration is uniform, the values of $\frac{s}{n^2}$ with a proper choice of units should be constant. If the time of vibration of the strip is known (it can be measured by timing say 50 vibrations with a stop watch if the vibrations are slow enough to count), the actual values of t can be calculated from the number of waves. Thus, if the strip makes five complete oscillations per second, AC, CE, EG, ... each represent one-fifth of a second. If the time of vibration cannot be determined, we may take it as an arbitrary unit of time for the experiment. AC will then represent 1 unit, AE 2 units, and so on. The absolute value of the time is only required if we wish to calculate the acceleration in cm. per sec. per sec.

The values of $\frac{s}{n^2}$ may be calculated as in the following example, or s may be plotted against n^2 .

Number of Complete Vibrations, n .	Distance Moved, s .	$\frac{s}{n^2}$
1	0.9 cm.	0.90
2	3.75 "	0.94
3	8.50 "	0.94
4	15.78 "	0.95
5	23.80 "	0.95
Mean (neglecting first observation)		0.945
$\frac{s}{n^2}$ is constant acceleration is uniform.		
Time of one vibration = $\frac{1}{10}$ sec.		
$\frac{s}{t^2} = 100 \frac{s}{n^2} = 94.5$		
Acceleration = $2 \frac{s}{t^2} = 189.0$ cm. per sec. per sec.		

EXPERIMENT 29.—To determine how the acceleration of the trolley varies with the slope of the plane.

Measure the acceleration of the trolley as in the previous experiment. Measure the slope of the plane as in Experiment 27 by measuring the vertical heights of two points on the track, a measured distance apart, above the level of the bench. If the two points are D cm. apart as measured along the track, and their vertical heights above the bench are d_1 and d_2 then $\frac{D}{d_1 - d_2}$ is the sine of the angle θ made by the track with the horizontal. The acceleration of the trolley down the plane can be shown to be $g \sin \theta$ where g is the acceleration due to gravity. The acceleration should thus be proportional to $\sin \theta$. Measure the acceleration a for different values of θ from 5° to 20° . Measure the corresponding values of $\sin \theta$, and determine the average value of $\frac{a}{\sin \theta}$. This should be constant and equal to g .

The acceleration of the fly-wheel on its inclined rails can also be shown to be proportional to $\sin \theta$, where θ is the inclination of the rails to the horizontal. The ratio in this case, however, is not equal to g , but is some fraction of it which depends on the radii of the wheel and its axle.

EXPERIMENT 30.—Determination of velocity from the space-time diagram.

If in the previous experiments we plot the space described by the fly-wheel or trolley against the time taken we obtain what is known as a *space-time diagram*. If the time is plotted along the horizontal axis the graph will curve upwards as shown in Fig. 21. Suppose we wish to determine the actual velocity of the body on passing some given point on its track represented by the point P on the graph. The velocity is the rate at which s is varying with t , at the point P . Let us take two points, p_1 and p_2 , one on each side of P , and draw p_1q parallel to the axis of t , and p_2q parallel to the axis of s . Then p_2q is the distance covered in the time p_1q , and the ratio $\frac{p_2q}{p_1q}$ is obviously the average velocity of the body between the two points p_1 and p_2 . But if we join p_1p_2 , and produce it

to meet the time axis R, this ratio is also equal to $\frac{p_1Q}{QR}$, that is, to the tangent of the angle p_1RQ . Now this average velocity obviously becomes more and more nearly the actual velocity at P, as p_1 and p_2 are taken nearer and nearer to P.

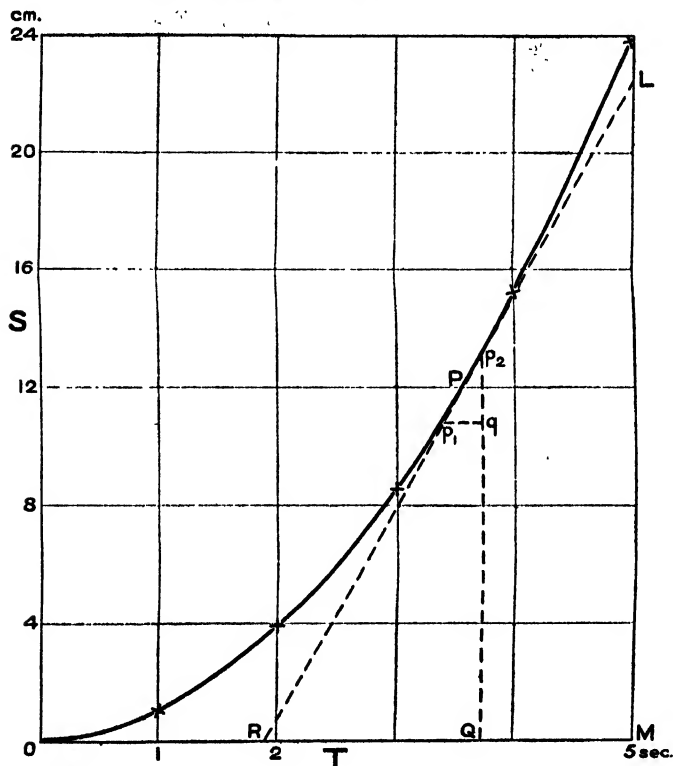


FIG. 21.—Space-time Diagram for the Motion of a Trolley on an Inclined Plane (Experiment 28).

But when they coincide with P, the line p_1p_2 becomes the tangent to the curve at the point P. Hence, to find the velocity of a body at any point on its track, construct the space-time diagram of the body, and draw a tangent to the graph at the given point. The tangent of the angle which

This line makes it possible to find the velocity of the body at any point on the curve. To find the velocity at a particular point, draw a tangent to the curve at that point without intersecting it. The vertical distance PQ and the horizontal distance QR can then be read off on the corresponding scales.

If the tangent is produced to the edge of the graph at L say (Fig. 21), the ratio $\frac{LM}{RM}$ is equal to the ratio $\frac{PQ}{RQ}$ and will therefore give the velocity at P . LM and RM are usually the most convenient measurements to make.

Plot, as described, a space-time curve for the fly-wheel from observations already made. Determine from the graph the velocity of the fly-wheel 1, 2, 3 . . . seconds from the beginning of its motion. Compare the velocities so determined with those deduced from the formula for uniform acceleration $v=at$, where v is the velocity after t seconds and a the acceleration.

In the same way plot a space-time curve for the motion of the trolley on the inclined plane. Determine the velocity of the trolley at a series of different distances from the starting-point, and verify that $v^2=2as$, where a is the acceleration.

§ 10. THE PARALLELOGRAM OF FORCES.

If two forces acting on a particle are represented in magnitude and direction by two adjacent sides of a parallelogram, the resultant of these forces is a single force represented in magnitude and direction by the diagonal of the parallelogram which passes through their point of intersection.

This important result may be verified experimentally in the following way :

EXPERIMENT 21.—To verify the principle of the parallelogram of forces.

A large drawing-board is clamped in a vertical position, and two ball-bearing pulleys are clamped one at each end of its

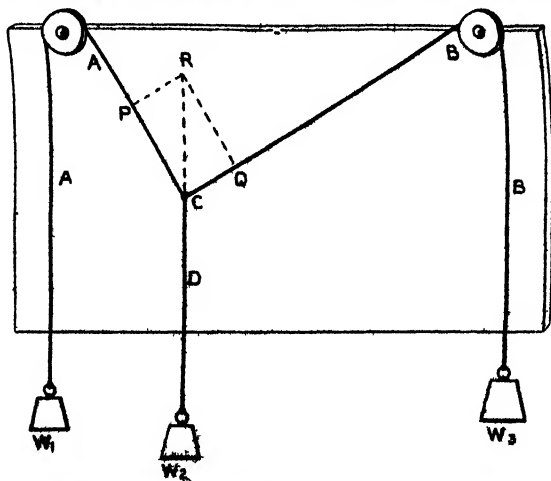


FIG. 22.—Experiment on the Equilibrium of Three Forces acting at a Point. The Parallelogram of Forces.

upper edge (Fig. 22). A long string is passed over both pulleys, and weights of say 3 lb. and 4 lb. are attached to

its ends. A second short string is knotted to the first about midway between the pulleys, and a third weight, of say 5 lb., is attached to this string. All the weights should then be hanging freely without rubbing against the board. Allow the system to come to rest, displacing it once or twice from its position of equilibrium to make sure that it always returns to the same position and that the pulleys are not sticking. Mark the position of each of the three portions of string on a sheet of white paper pinned to the drawing-board, by making a couple of pin pricks at some distance apart, immediately behind each portion of the string. Remove the weights and draw the lines AC, BC, and DC, which are the positions of the strings. From C mark off a distance CP along the line CA proportional to the weight W_1 which was attached to this string. A scale of 1 inch to 1 lb. will be convenient if the weights have the values suggested, but any scale which will give a good-sized diagram may be employed, say for example 5 cm. to the lb. The scale adopted should be written on the diagram in all cases, and, of course, the same scale must be strictly adhered to throughout the same diagram.

In the same way mark off a distance CQ along CB to represent the weight W_2 on this string. Complete the parallelogram CPRQ and draw the diagonal CR. If the parallelogram law is true CR should be in the same straight line as the string CD, which is the *direction* of the third force, and its length should be equal to the weight W_3 hanging on this string, on the scale of the diagram.

The following experiment will serve to illustrate the application of the parallelogram law :

EXPERIMENT 32.—To find the weight of a body by the funicular polygon.

Cover a drawing-board with a sheet of white paper and drive two stout nails into the board, one near each of the top corners (Fig. 23). Clamp the board in a vertical position and tie a string securely to each of the nails so that it hangs quite loosely between them. If it is desired not to drive nails into the board the string may be tied to two retort stands, one on each side of the board so that the string hangs in front of the board. At about one-third of the length of the string from each end knot on another string. To one of these attach the unknown weight X, and to the other a suitable known weight

W. The strings will then hang as shown in Fig. 23. This is known as a funicular polygon (from the Latin word *funis* = rope). The position of each of the strings is marked on the paper as in the previous experiment.

To calculate X proceed as follows. From a point A on the line AB draw a line parallel to BE and mark off along this line a length AP to represent the known weight W on some suitable scale. If W is 500 gm. we may take a scale of 1 cm. = 50 gm. in which case AP would be 10 cm. The scale should be as large as the paper will permit. From P

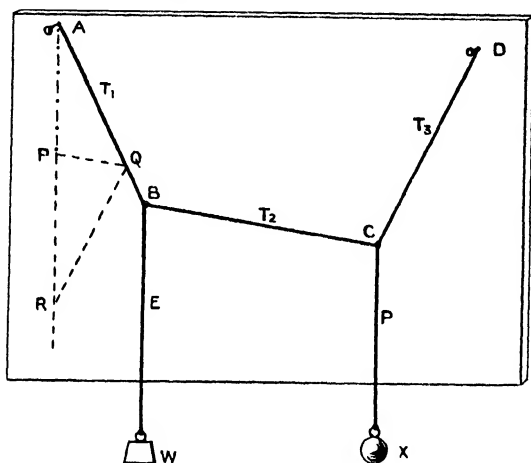


FIG. 23.—Determination of an Unknown Weight by the Funicular Polygon.

draw a line PQ parallel to BC and meeting AB, produced if necessary, in Q. Now the point B is in equilibrium under the action of the weight W and the tensions T_1 and T_2 in the strings. Now in the triangle APQ the three sides are parallel to these three forces and the side AP is equal to W on the scale of the diagram. Hence, by the triangle of forces, AQ represents the tension T_1 and PQ the tension T_2 . From Q draw a line QR, parallel to the string CD and meeting the vertical line AP produced in R. Now the point C is maintained in equilibrium by the tensions T_2 and T_3 and by the unknown weight X. These forces are parallel to the three sides of the triangle PQR, while the side PQ is further equal

to the tension T_2 . Hence the side PR is equal to the unknown weight X, on the scale of the diagram. Thus if W is 500 gm. and AP is 10 cm. so that 1 cm. represents 50 gm., and if by measurement PR is found to be 14.24 cm., the unknown weight X is equal to $14.24 \times 50 \text{ gm.} = 712 \text{ gm.}$

Very careful drawing is required if the result is to be at all accurate. The parallel lines should be drawn with a fine-pointed pencil, by means of a straight edge and a good set square.

§ 11. PRINCIPLE OF MOMENTS

THE moment of a force about a given point is the product of the magnitude of the force and the length of the perpendicular from the point upon the line of action of the force. It represents the tendency of the force to produce rotation round the point. If a body is in equilibrium under the action of a system of forces it is obvious that the algebraical sum of the moments of all the forces about any point must be zero. The moment is usually considered positive if it tends to produce rotation in a counter-clockwise direction, and negative if the rotation is clockwise.

EXPERIMENT 33.—To verify the law of moments for forces meeting at a point.

Set up the apparatus used in Experiment 31 and transfer the direction of the strings to the paper as described. Take any point in the paper (preferably a point not too near the

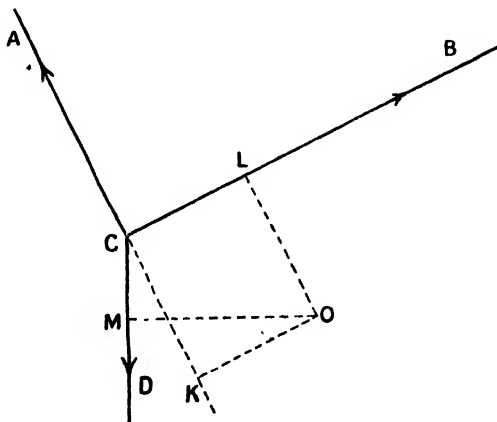


FIG. 24.—Verification of the Principle of Moments for Forces meeting at a Point.

line of action of any of the forces) and drop perpendiculars OK, OL, OM on each of the lines of action of the forces. Measure accurately the length of each of these perpendiculars and multiply by the corresponding force. Note whether the rotation produced by the force would be counter-clockwise or clockwise and if the latter give the value a negative sign. Find the algebraical sum of the three products. The result should be zero.

Example (see Fig. 24) :

Direction of Force.	Force.	Length of Perpendicular from O.	Moment about O.
CA	600 gm. wt.	9.82 cm.	- 5892 gm. cm.
CB	800 "	9.54 "	- 3568 "
CD	1000 "	4.46 "	+ 9540 "
Algebraical sum = +			80 gm. cm.

The algebraical sum of the moments is zero within the limits of experimental error.

The principle of moments is of great importance in the case of a system of parallel forces.

EXPERIMENT 34.—To verify the law of moments for a system of parallel forces.

A metre scale has a small hole carefully drilled through its middle point (*i.e.* the 50 cm. mark), and is suspended by a piece of thin twine passing through the hole and attached to a retort stand (Fig. 25). Should the scale be found not to balance exactly about this point of suspension it can be made to do so by loading the lighter side with a small piece of wire. If the hole is drilled rather nearer the top edge than the bottom edge of the scale the latter can be made to balance in a horizontal position. (If the hole is nearer the bottom edge the scale will tend to overturn.) Since the scale is suspended at a point immediately above its centre of gravity its weight acts through the point of support and thus has no moment about this point. We may, therefore, neglect it in our calculations.

Take two weights, say a 500 gm. weight and a 200 gm. weight, and fasten a loop of cotton to each so that they can slide along the metre scale. Place one on each arm of the scale and adjust their positions so that the scale balances horizontally. Record the positions. Move one of the

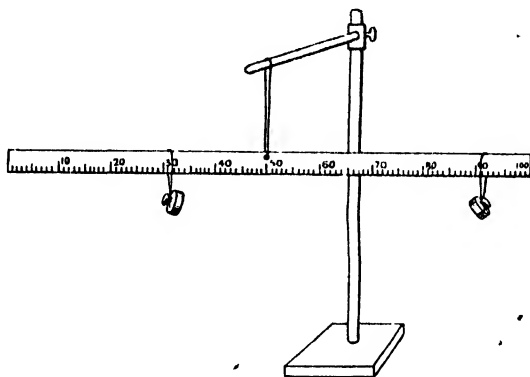


FIG. 25.—Verification of the Law of Moments for a System of Parallel Forces.

weights a few centimetres and again adjust the other weight until a balance is obtained. Continue until a series of five or six observations have been made. In each case the product of the weight into its distance from the point of suspension should be the same for the two weights. Record as in the following example :

Distance from Point of Support.		Moment about Point of Support.		Algebraical Sum of Moments.
500 gm.	200 gm.	500 gm.	200 gm.	
5.0 cm.	12.40	2500 gm. cm.	- 2480 gm. cm.	+ 20 gm. cm.
7.5 "	18.75	3750 "	- 3750 "	0 "
10.0 "	25.00	5000 "	- 5000 "	0 "
12.5 "	31.20	6250 "	- 6240 "	+ 10 "
15.0 "	37.45	7500 "	- 7490 "	+ 10 "
The sum of the moments is zero within the limits of experimental error.				

The experiment can be extended by using more than two weights. When the scale has been adjusted to equilibrium the algebraical sum of the moments of all the weights about the point of support should be zero.

EXPERIMENT 35.—Given a metre scale and a 200 gm. weight, determine the weight of the given body.

If one of the two weights is unknown, its weight can be found by the method of the previous experiment, by adjusting the positions of the known and unknown weights as in the previous experiment until the scale is horizontal, and applying the principle of moments. This is the principle of the common steelyard, which is often used for weighing heavy bodies. For accuracy three independent settings of the weights should be made and the mean taken. With care it should be possible to obtain an accuracy of at least 1 per cent.

§ 12. CENTRE OF GRAVITY

THE centre of gravity of a body is a point fixed relatively to the body at which the whole weight of the body may be supposed to act.

If a body is freely suspended from a support, it will come to rest with its centre of gravity vertically below the point of support. We may use this result to determine the centre of gravity of a flat plate of any shape.

EXPERIMENT 36.—To determine the centre of gravity of a given flat plate.

The plate must be capable of being suspended freely from at least two different points. This may be done either by making two small holes at different points near the edge of the plate, and tying loops of string through the holes, or by making somewhat larger holes so that they may be passed over a smooth horizontal nail or peg as shown in Fig. 26.

Suspend the body from one of the holes, A, and tie a plummet to the point of support so that it hangs just in front of the plate without actually touching it. Make two marks on the plate immediately behind two points on the plumb line, taking care not to displace either the plate or the plumb line in the process, and rule a straight line on the surface passing through these two points. The centre of gravity of the plate must lie on this line. Now suspend the plate from another point, such as D, and repeat. The centre of gravity must lie at the point where the two lines cross, *i.e.* at O on the

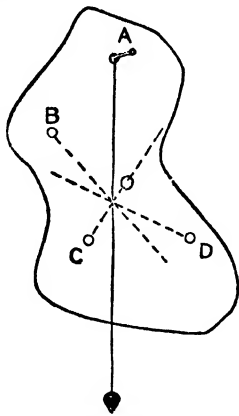


FIG. 26. — Experimental Determination of the Centre of Gravity of a Flat Plate.

diagram. For accuracy the two lines should meet as nearly as possible at right angles. As a verification the plate may be suspended from other points such as B and C. All the lines drawn should pass through the same point O, which is the centre of gravity of the plate.

In the case of symmetrical bodies the position of the C.G. can often be calculated. Thus the C.G. of a rectangle is at the point of intersection of its diagonals, and that of a triangle at the point of intersection of the lines joining the points of the triangle to the mid points of the opposite sides. Any plane figure which is bounded by straight lines can be divided into a number of rectangles and triangles. The centre of gravity can then be determined by the method illustrated in the following experiment :

EXPERIMENT 37.—To determine the centre of gravity of a thin plate bounded by straight lines.

On a sheet of stiff cardboard draw a square (ABCD, Fig. 27) with a side of 10 cm., and on one of the sides BC

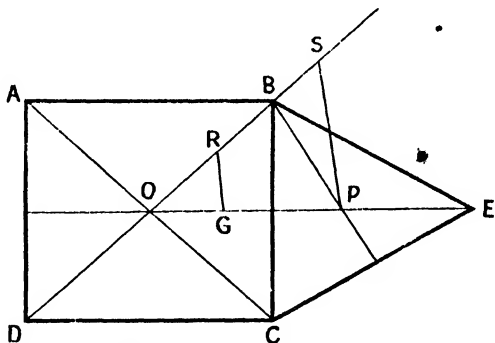


FIG. 27.—Determination of the Centre of Gravity of a Flat Plate of Regular Shape.

erect an isosceles triangle BCE. Determine the C.G. of the figure so formed as follows. Draw the two diagonals of the square meeting in O. Join EO, which will bisect the side BC of the square, and join B to the middle point of the side CE. If these lines intersect in P, P is the C.G. of the triangle. Suppose that G is the C.G. of the whole figure. The weight

of the square acts at O and that of the triangle at P. Since the whole figure must balance about G the moments of these two weights about G must be the same. But if the cardboard is uniform the weights of the figures are proportional to their areas. Hence $(\text{area of square}) \times OG = (\text{area of triangle}) \times GP$. Since $OG + GP = OP$, which can be measured, we can calculate the position of G, or we can find it graphically as follows. Produce OB and mark off along it a distance OR proportional to the area of the triangle, and a second length RS proportional to the area of the square. Join S to P and draw RG parallel to SP meeting OP in G. Then

$$\frac{OG}{GP} = \frac{OR}{RS} = \frac{\text{Area of triangle}}{\text{Area of square}}$$

$\therefore OG \times (\text{Area of square}) = GP \times (\text{Area of triangle})$
and therefore G is the centre of gravity of the whole figure.

Verify the result by carefully cutting out the figure and finding its C.G. by the method of the previous experiment.

§ 13. FRICTION

IF a block of wood is resting on a horizontal surface it is found that a definite force must be applied to the block before it will begin to move. If a smaller force is applied no motion will take place. The motion of the block is thus resisted by a force known as friction. The maximum resistance offered by friction is known as the limiting friction between the surfaces. It is found that the limiting friction is directly proportional to the reaction between the two surfaces. The ratio of the limiting friction to the reaction is thus a constant for the same pair of surfaces, and is known as the coefficient of friction for the surfaces.

EXPERIMENT 38.—To show that the limiting friction is proportional to the reaction, and to find the coefficient of friction.

A wooden tray A (Fig. 28), about $12 \times 6 \times 4$ cm., has its lower face planed, and a small eye is screwed into the centre

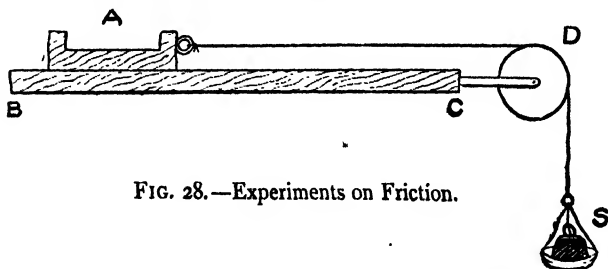


FIG. 28.—Experiments on Friction.

of one end. An accurately planed board, BC, about 30 cm. long and 10 cm. wide, has a ball-bearing pulley, D, at one end, the height of the pulley being adjusted so that a string fastened to the eye in the tray and passing over the pulley will be horizontal. A scale pan, S, is attached to the end of the string which hangs down over the edge of the bench, as shown in the figure. The scale pan and the tray are each weighed.

The apparatus is set up, and weights are added to the scale pan until on tapping the board the tray will just begin to slide along it. Slight projections on the surfaces may interfere with the motion, so that the experiment should be tried several times, and the average value of the weight determined.

An additional weight is placed on the upper surface of the tray and the experiment repeated. This process is repeated with three or four different weights. The sum of the weights of the pan and its contents is the force which just overcomes the limiting friction. The weight of the tray and its contents is the reaction between the surfaces. The results may be recorded as follows :

Surfaces used : Wood on wood.				
Weight of tray = 225 gm.				
Weight of pan = 52 ,,				
Weight on Tray.	Weight in Pan which just produces Motion.	Reaction R.	Limiting Friction F.	$\frac{F}{R}$.
0 gm.	21 gm. wt.	225 gm. wt.	73 gm. wt.	0.32
200 ,,	84 ,,	425 ,,	136 ,,	0.32
400 ,,	143 ,,	625 ,,	195 ,,	0.31
600 ,,	208 ,,	825 ,,	260 ,,	0.31
Mean coefficient of friction, wood on wood = 0.315				

By placing a flat sheet of glass or metal on the board, the coefficient of friction between wood and glass or wood and metal can be investigated in the same way. The lower surface of the block can also be varied by covering it with cartridge paper or leather. The effect of soaping or oiling the surfaces can also be studied.

If a block rests on a plane and the angle between the plane and the horizontal is gradually increased, an angle is eventually reached at which the block will begin to slide down the plane. The maximum angle at which the block can rest on the plane without sliding down it is called the angle of friction. It can be shown that if θ is the angle of friction, $\tan \theta =$ the

coefficient of friction between the surfaces. For resolving the weight W of the block perpendicular and parallel to the plane, the reaction R is equal to $W \cos \theta$, while the force down the plane is $W \sin \theta$. When the block is on the point of sliding the latter is obviously equal to the limiting friction. Hence the coefficient of friction $= \frac{F}{R} = \tan \theta$. θ is thus independent of the weight of the block.

EXPERIMENT 39.—To determine the angle of friction for a wood block resting on a wooden plane.

The block and plane used in the previous experiment can be employed. The string is detached from the block which is placed resting on the plane. One end of the latter is placed against some solid obstacle, and the other end is gradually elevated by placing a wooden block under it and gradually moving the block along towards the fixed end. A hinged plane such as that used in Experiment 28 is, of course, more convenient.

The plane is gradually tilted, and gently tapped, until the angle is reached at which the block just commences to slide when the plane is *gently* tapped. The tangent of this angle can be measured directly by measuring the vertical height of the surface of the plane above the level of the table at two points, the *horizontal* distance apart of which is also measured. The difference between these two heights divided by the horizontal distance is equal to the tangent of the angle of the plane, and hence to the coefficient of friction between the surfaces.

By placing different weights on the block it can be shown that the angle of friction is independent of the weight of the block.

§ 14. HYDROMETRY

SOME experiments in hydrostatics, *e.g.* the determination of specific gravities by the density bottle, by the U-tube and by Archimedes' principle have been already described in the chapter on Practical Measurements. (*See Experiments 16-23.*)

The principle of Archimedes may be applied also to floating bodies. A floating body will take up such a position that the weight of the volume of liquid which it displaces is equal to the weight of the body itself. This result may be used for comparing the densities of liquids.

EXPERIMENT 40.—To compare the densities of water and salt solution. Principle of the variable immersion hydrometer.

Take a thin-walled test-tube and place a few lead-shot in it, so that it will float upright on water. Place a paper or cardboard scale inside the test-tube, securing it with a little soft wax, and float the tube in a gas jar containing water. If necessary, place more lead-shot in the tube until the top of the tube is about 1 inch above the surface of the water. Then note the reading of the surface of the water on the scale inside the test-tube.

Now float the test-tube in a second jar containing salt solution, and again note the level of the surface of the liquid on the scale in the test-tube.

Since the weight of the test-tube and its contents is constant, the weight of water displaced by the floating tube is equal to the weight of salt solution displaced. If V is the volume of water displaced, V' the volume of solution, while ρ_0 and ρ are the corresponding densities, we have

$$V\rho_0 = \text{weight of test-tube and contents} = V'\rho.$$

The volumes displaced are proportional to the lengths of tube immersed. These lengths can be found by measuring from the corresponding scale divisions to the bottom of the

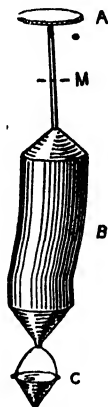
taken out of the water, dried on a clean duster and weighed, with its contents.

It is then floated on the salt solution, and more shot is added until it is again immersed to exactly the same mark as before. It is then again dried and weighed. Since the tube floats in exactly the same position in both cases the volumes of the liquids displaced are the same. Let W_0 and W be the weights of the tube and contents when floating on water and salt solution respectively. Then

$$W_0 = V\rho_0; \quad W = V\rho;$$

$$\frac{\rho}{\rho_0} = \frac{W}{W_0}$$

This principle is applied in Nicholson's hydrometer. This usually takes the form shown in Fig. 29. A scale pan A, at the upper end of a short metal stem, is carried by a cylindrical float B, to the lower end of which is attached a second scale pan C, which is loaded so that the instrument floats in an upright position. Masses are added either to A or C according to circumstances until the hydrometer floats with a mark M engraved on the stem on the surface of the liquid. The volume of liquid displaced by the hydrometer is therefore always the same.



EXPERIMENT 42.—To determine the specific gravity of a liquid by Nicholson's hydrometer.

Weigh the hydrometer ($=W$). Float it on water in a gas jar and add weights W_1 to the upper pan A, until it sinks to the mark M. Now float the instrument on the liquid whose specific gravity is required, and again load A until the hydrometer is immersed to M. Let W_2 be the weight required.

FIG. 29.—
Nicholson's
Hydrometer.

The specific gravity of the liquid is $\frac{(W + W_2)}{(W + W_1)}$

EXPERIMENT 43 —To find the specific gravity of a solid by Nicholson's hydrometer.

The hydrometer can also be used to determine the specific gravity of a solid. In this case it floats on water during the

whole experiment, and in fact merely serves as a not very sensitive balance.

The hydrometer is floated on water and the weights W_1 which must be placed on the pan A to immerse the instrument to the fixed mark M are determined. The weights are removed and the solid is placed on the pan A, and weights W_2 are again placed in this pan until M is again on the surface of the water. $W_1 - W_2$ is obviously the weight of the solid. The solid is now transferred to the lower pan C, so that it is completely immersed in water, and weights W_3 are placed in the pan A until the hydrometer is in adjustment. Owing to the upthrust of the water on the solid W_3 will be greater than W_2 , and $W_3 - W_2$ is obviously equal to the upthrust of the water on the solid, and is thus, by Archimedes' principle, equal to the weight of water displaced by the solid.

$$\begin{aligned} \text{Hence the specific gravity of the solid} &= \frac{\text{wt. of solid}}{\text{wt. of water displaced}} \\ &= \frac{W_1 - W_2}{W_3 - W_2} \end{aligned}$$

Example:

	<i>Weights in pan</i>
With no solid	= 115.5 gm.
With solid in upper pan =	42.8 "
With solid in lower pan =	52.4 "
Specific gravity of solid (iron) =	$\frac{115.5 - 42.8}{52.4 - 42.8}$
	= 7.6

§ 15. MEASUREMENT OF PRESSURE

THE pressure at a point in a fluid is measured by the average force per unit area, measured over a very small area containing that point. Pressure is measured in dynes per square centimetre in the absolute C.G.S. system of units. For practical purposes it is more often expressed in gravitational units, *e.g.* in grams per sq. cm. or in pounds per square foot. Another way of measuring pressure is by the vertical height of the column of mercury which the pressure would support. Thus the pressure of the atmosphere is almost always stated as being equal to so many centimetres or millimetres of mercury. It can be shown that the pressure due to a liquid of density ρ at a depth h below its free surface is equal to $h\rho$ in gravitational units of pressure, or to $h\rho g$ in absolute units. It is thus directly proportional to the depth.

EXPERIMENT 44.—To show that the pressure due to a liquid is directly proportional to the depth.

A long cylindrical *thin-walled* brass tube closed at one end may be used. It is weighed and then floated on water in a tall gas jar, the upper end of the tube being passed through the ring of a retort stand so that the tube floats in a vertical position. The length of tube immersed is measured by means of a metre scale. A known weight, say 200 gm., is placed inside the cylinder, which sinks deeper into the water. The new depth of immersion is measured. This is continued until the cylinder is almost completely immersed. The total weight to be supported at any stage is the weight of the empty cylinder plus the weights it contains. If the cylinder is floating in a vertical position this weight is supported by the force exerted by the water on the lower end, since the force on the vertical sides, being horizontal, will have no component in the vertical direction. If P is the pressure at the lower end of the tube and α the area of cross-section of the tube, the thrust on the lower end due to the pressure of the

water will be $P\alpha$. This must equal the weight of the cylinder and its contents. Hence P is proportional to W . Plot a curve between W and d where d is the depth of the lower end of the cylinder below the surface of the water. The graph should be a straight line.

The pressure at a point is generally measured by finding the length of column of some standard fluid which the pressure will support. Water and mercury are the liquids most frequently used, but concentrated sulphuric acid and heavy oil are sometimes used for special purposes. Since mercury is 13.5 times as dense as water, a pressure which would support a column of mercury 1 cm. long would support a column of water 13.5 cm. long. The water gauge is thus more sensitive than the mercury gauge.

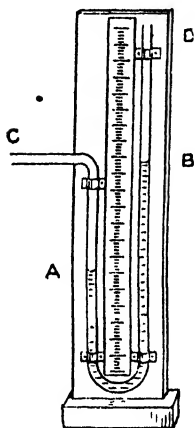


FIG. 30.—Siphon Manometer.

A very common type of gauge consists of a glass U-tube, one end being open to the air, and the other end connected to the pressure it is wished to measure. The standard liquid occupies the lower half of the U (Fig. 30). If the pressure to be measured is greater than the atmospheric pressure, the surface of the liquid in the open limb will be higher than that in the other. The difference in level in the two limbs measures the difference between the pressure to be measured and the atmospheric pressure. The latter can be obtained by reading the barometer. If the liquid in the gauge is mercury, the pressure at C is equal to the height of the barometer, plus the height of the column AB, in centimetres or millimetres of mercury. If the liquid in the gauge is water, the distance AB must be divided by the density of mercury to reduce it to centimetres or millimetres of mercury, or, of course, the pressure may be expressed in centimetres of water by multiplying the height of the barometer by the density of mercury.

If the level of A is above the level of B, the height AB must, of course, be subtracted from the barometric height.

EXPERIMENT 45.—To make a siphon manometer and to measure the pressure of the gas supply.

Take a piece of glass tubing about 50 cm. long and about $\frac{1}{2}$ cm. in diameter. Bend it in the middle into the form of a U in a fish-tail burner, and bend one end out as shown in Fig. 30. Fill the lower half of the tube with water, which may be coloured pink, if desired, by adding a trace of potassium permanganate. If the gauge is to be a permanent one, it may be secured to an upright stand. For the present experiment it will be sufficient to clamp the U-tube, with a half-metre scale behind it, in a wooden clamp, so that the tube and the scale are vertical.

Attach the end C to the gas tap by a piece of rubber tubing and slowly turn on the tap. Read off the positions of the two surfaces A and B on the scale. The difference AB is the length of a column of water which will just balance the difference in pressure between the gas supply and the atmosphere. Record the barometric height. The result may be calculated as follows :

Barometric height = 754.25 mm. of mercury

Manometer readings :

Open limb = 18.6 cm.

Pressure limb = 13.2 cm.

Difference = 5.4 cm.

Pressure of gas supply is greater than atmospheric by 5.4 cm. of water ; or

$$\frac{5.4}{1.35} = 0.385 \text{ cm. of mercury.}$$

Hence actual pressure of gas supply

= 754.25 + 3.85 mm. of mercury

= 758.10 mm. of mercury.

§ 16. BOYLE'S LAW

THE pressure of a given mass of gas at constant temperature is inversely proportional to the volume.

This law can be verified very simply as follows :

EXPERIMENT 46.—To verify Boyle's Law. (Simple method.)

A capillary glass tube about 30 cm. long, and with a bore of about 1 mm. diameter, is sealed at one end, and a mercury index about 8 cm. in length is introduced, so as to enclose a column of air between the closed end of the tube and the index. The index can be introduced by heating the tube gently, and holding it with the open end beneath the surface of mercury.

The tube, when it has cooled down to the temperature of the room, is placed horizontally on the bench. The length of the mercury column is measured, and also the distance between the sealed end of the bore and the nearer end of the mercury index. The tube is then clamped in a vertical position, with the open end upwards, and the length of the enclosed air column again measured. The tube is then inverted, so that the open end is downwards, and the length of the enclosed air column again measured. The barometer is then read.

When the tube was horizontal the pressure of the gas inside the tube was equal to that of the atmosphere. When the open end was uppermost, the pressure on the enclosed air was equal to that of the atmosphere, plus the pressure due to the mercury index ; while, with the tube inverted, the pressure on the enclosed air was less than atmospheric by the pressure due to the index, since the atmosphere, acting on the lower surface of the index, had to support the pressure of the index, in addition to the pressure of the gas, on its upper end. The three pressures are thus known, while the corresponding volumes of the enclosed gas are proportional to the length of tube occupied. The results may be recorded as follows :

Height of barometer = 752.8 mm. = 75.28 cm. Length of mercury index = 8.24 cm.			
Tube.	Length of Enclosed Air Column = V.	Pressure on Enclosed Air = P.	P . V.
Horizontal . . .	14.50 cm.	75.28 cm.	1092
Vertical (open end up) .	13.10 „	75.28 + 8.24	1094
„ (open end down)	16.35 „	75.28 - 8.24	1095

The total change in pressure in the previous experiment is only small. For more extended observations we require more elaborate apparatus. The following method, due to Jolly, gives good results :

EXPERIMENT 47.—To verify Boyle's Law.

A glass tube AB (Fig. 31), about 30 cm. long and about $\frac{1}{2}$ cm. diameter, is sealed at one end, and the other end is drawn out so that it can be attached to a piece of rubber pressure tubing about 1 metre in length. A second piece of glass tubing, C, about 1 cm. in diameter, is attached to the other end. The first tube is clamped vertically on a vertical board, while the second piece is attached to a slide which moves up and down a vertical rod, and can be clamped in any required position. Mercury fills the rubber tubing and the lower parts of the glass tubes, while a vertical metre scale screwed to the board enables the position of the two mercury surfaces to be read. The apparatus can, of course, be rigged up on retort stands if a proper stand is not available. It will be convenient if the mercury is adjusted so that the level of the mercury is the same in both tubes

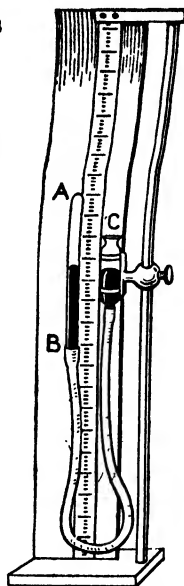


FIG. 31.—Apparatus for verifying Boyle's Law.

when the mercury stands about half-way up the closed tube.

To begin the experiment, adjust the position of the sliding tube C until the mercury is exactly at the same level in both tubes. In recording the position of mercury in a tube it is usual to read the top of the meniscus. The two columns of mercury now exactly balance each other, and the pressure of the gas enclosed in the tube is, therefore, equal to that of the atmosphere P. This is obtained by reading the barometer. Measure the length of the enclosed column of air. As the tube is uniform the volume of the enclosed air is proportional to the length.

Now raise the open tube. The mercury will rise in both limbs, but more rapidly in the open than in the closed limb. Let h be the *difference* in level between the mercury surfaces. The pressure on the enclosed gas is then equal to $P + h$ cm. of mercury. Measure the corresponding length of the enclosed column of air. Take a series of such readings until the open tube is as high as it will go.

Now lower the open tube until it is below its original position. The level of the mercury in the open tube will then be below that in the closed tube. The pressure on the enclosed air is then equal to the atmospheric pressure, less the difference in levels between the two mercury surfaces. Continue the observations at intervals until the open tube will go no lower.

The truth of Boyle's law for the enclosed gas can be investigated by multiplying the pressure into the corresponding length of the enclosed column of gas. By Boyle's law, this product should be constant. A graph should also be plotted between the pressure and the *reciprocal* of the volume of the enclosed gas. The graph should be a straight line, since the pressure is *inversely* proportional to the volume, and hence directly proportional to the reciprocal of the volume. A graph plotted between the pressure and the volume, directly, is quite useless for verifying Boyle's law. It is a portion of a curve known as a rectangular hyperbola, but as there are no ready means of identifying such a curve, or of observing departures from it, it is useless for our purpose. The following record of an experiment with a Boyle's law apparatus indicates the most accurate method of making and recording the measurements ;

Height of barometer = 75.6 cm.					
Scale reading of closed end of gas tube = 60.0 ,,					
Mercury Levels.		Difference in Level, h $= b - a$.	Length of Air Column V $= (60.0 - a)$.	Pressure, P $= 75.6 + h$.	P. V.
Closed Tube $= a$.	Open Tube $= b$.				
45.2 cm.	45.2 cm.	0	14.8	75.6	1117
47.0 ,,	57.2 ,,	10.2	13.0	85.8	1112
48.6 ,,	71.1 ,,	22.5	11.4	98.1	1116
50.0 ,,	86.3 ,,	36.3	10.0	111.9	1119
41.6 ,,	37.0 ,,	-14.6	18.4	61.0	1120
36.5 ,,	8.3 ,,	-28.2	23.5	47.4	1112
<p><i>Conclusion.</i>—P. V is constant within limits of experimental error.</p> <p>This is confirmed by plotting P against $\frac{1}{V}$. Resulting graph is a straight line.</p>					

ADDITIONAL EXERCISES AND EXAMINATION QUESTIONS.—II

1. Make experiments to investigate the motion of a small glass bead dropping through a column of glycerine.

(The glycerine may be contained in a graduated glass jar. Measure the time taken by the bead to describe different distances, and plot a space-time curve. After the first centimetre or so it will be found to be a straight line.)

2. Determine how the horizontal force required to deflect the bob of the given pendulum varies with the tangent of the angle of deflection.

3. Find the force required to move the given block of wood (a) up, (b) down the given plane when inclined at an angle of 10° to the horizontal. Record the weight of the block.

(The force can be measured by passing a string attached to the block over a pulley clamped at the end of the plane, and adding weights to the scale pan.)

4. Determine how the ratio of the force which must be applied to the given trolley, to keep it at rest on the, given

inclined plane, varies with the sine of the angle between the plane and the horizontal.

(Use the same apparatus as in the previous example, substituting a trolley for the wood block.)

5. Three strings are knotted together. Two of them pass over pulleys and have weights attached, so that they keep in equilibrium an unknown weight attached to the third string. Find the value of the unknown weight.

6. Knot three strings together at a point. Attach equal weights to two of them, and pass these over two pulleys in the same horizontal line. From the third string suspend a scale pan. Place different weights in the pan and measure the corresponding angle between the two strings passing over the pulleys. Plot a graph between the total weight carried by the third string, and the cosine of half the angle between the other two strings.

7. On a sheet of cardboard draw a square ABCD of side 10 cm. and its diagonals intersecting in O. Determine the centre of gravity of the figure ABOCD obtained by removing the triangle BOC, (a) by calculation, (b) by direct experiment.

8. You are given a metre scale, a piece of string, and a 100 gm. weight. Determine the weight of the scale.

(Tie the weight near one end of the scale, and balance the scale on a loop of string until it is in equilibrium. Take moments about the point of support. The weight of the scale acts through its C.G., which may be determined by balancing the scale by itself.)

9. Assuming the accuracy of Boyle's law, make experiments with the given Boyle's law apparatus to determine the pressure of the atmosphere.

(Plot a graph between h (see Experiment 47) and $\frac{1}{V}$.)

Produce the straight line back to the axis of P. The intercept on this axis between the zero and the point where the graph cuts it is the atmosphere pressure. The student is advised to make sure that he understands why this is so.)

BOOK III

HEAT

§ 17. THE MEASUREMENT OF TEMPERATURE

THE temperature of a body, that is to say, its hotness or coldness, may be estimated roughly by the sense of touch. We can determine directly which is the hotter of two beakers of water if they differ in temperature by more than a few degrees. The method is not very sensitive, and in any case it affords no basis for the measurement of temperature. Temperature cannot be measured directly in the way that we can measure mass or length. In practice, therefore, temperature is measured by selecting some property, such as the volume of a given mass of liquid or gas, the length of a given bar, or the resistance of a given conductor, which can be measured directly, and which changes in a uniform way with change in temperature. The change in this property is then used as a measure of the corresponding change in temperature. The property generally employed is the expansion of some suitable fluid, generally mercury, enclosed in a suitable glass vessel. Such an instrument is known as a thermometer.

EXPERIMENT 48.—To construct a thermometer.

A capillary glass tube of about $\frac{1}{2}$ mm. bore with a small bulb blown on one end will be required. Owing to the fineness of the bore it will not be found possible to pour liquid directly into the bulb. To fill the thermometer, therefore, we proceed as follows. A small glass funnel is connected to the open end of the tube by means of a short piece of rubber tube (Fig. 32). If the two tubes are pushed well into the rubber so that their ends touch, the funnel will stand upright on the thermometer tube without falling over. Pour into the funnel rather more than sufficient methylated spirit to fill the thermometer completely, and

then gently heat the bulb with a bunsen flame. This will cause the air in the bulb to expand, and some will be expelled and can be seen bubbling out through the liquid in the funnel. Now allow the bulb to cool. As the air in the bulb cools and contracts some of the spirit will run into the bulb to take the place of the expelled air. The thermometer could be completely filled by repeating this process a sufficient number of times, but the process is a slow one. When, therefore, the



FIG. 32.—
Method
of filling
a Ther-
mometer
Tube.

bulb is about one-third full of liquid we heat it over a small flame until the spirit begins to boil. Allow it to boil vigorously until nearly all the liquid has boiled away. The rush of vapour carries out with it nearly all the air in the thermometer, and when the vapour is allowed to condense by taking away the flame the liquid in the funnel will run back into the bulb, completely filling it. If necessary the operation is repeated until the bulb is full, but with a little experience the bulb can be filled with one operation.

Occasionally, however, a small bubble persists in the bulb even after repeated heating and cooling. It should be shaken into the capillary stem, and can then often be removed by poking it with a fine wire passed down the tube. If not, the thermometer must be cooled in a stream of water from the tap, or better still, in a freezing mixture, and then heated gently by placing the bulb in a beaker of water at about 70°C . If the bubble has been shaken well into the stem the expansion of the liquid will drive it upwards through the capillary and out into the liquid in the funnel.

The process has been described in detail as it is often required for filling vessels with a narrow neck.

To complete the thermometer, place it in a vessel of water at a temperature below the boiling-point of the spirit and slightly above the highest temperature which it is desired to read with the thermometer. If methylated spirit is used for the thermometer this temperature should not be above 70°C . When the thermometer has attained the temperature of the bath slip off the funnel and the rubber tube. The thermometer will now be completely filled with liquid at the temperature of the bath. If it is desired to keep the thermometer for future

use, the liquid may be allowed to cool a little and the top of the capillary tube closed by heating it with a blow-pipe flame.

The apparatus now forms a simple alcohol thermometer. When the bulb is heated the column of liquid rises in the tube; when cooled the column falls. Each position of the upper end of the column in the stem corresponds to a single definite temperature.

NOTE.—*Methylated spirit is inflammable. The student should substitute water for methylated spirit until he has obtained some facility in the operations, though water, owing to its peculiar behaviour is unsuitable for a thermometric liquid.*

In actual thermometers mercury is generally employed instead of alcohol or spirit, and the tube is sealed while full of mercury so that there is a vacuum above the mercury column. Mercury is much more difficult to manipulate than spirit, as it boils at a much higher temperature, and the rush of cold mercury into the hot bulb is almost certain to crack the glass. To obviate this the mercury in the funnel is kept heated almost to boiling-point, the funnel being actually sealed on to the capillary stem. These operations require a considerable amount of experimental skill.

The instrument just made constitutes a simple thermometer but it is not yet furnished with a scale. An arbitrary scale of temperature could be constructed by making a convenient number of scratches at equal intervals along the stem. To make the readings of general use, however, we require a scale constructed according to some recognised plan. Two fixed temperatures are selected which can be readily reproduced when required and which are known to be constant, and the height of the column of liquid at these two temperatures is marked on the stem. The volume of the bore of the tube between the two marks is then divided into some suitable number of equal parts by a series of marks on the stem. If the bore of the tube is uniform, which can be tested before the thermometer is filled (see Experiment 14), this can be done by dividing the length of the stem between the two graduations into a corresponding number of equal parts. The fixed points always used in thermometry are (*a*) the melting-point of pure ice, and (*b*) the boiling-point of pure water under normal atmospheric pressure. On the Centigrade scale, generally employed for scientific work, the former is

marked 0°C. , and the latter 100°C. , the space between being divided into 100 equal parts or degrees. As alcohol boils at 78°C. the upper fixed point cannot be directly determined on our alcohol thermometer.

The method of determining the fixed points of a thermometer can be illustrated most conveniently with an ordinary graduated mercury thermometer (Fig. 33). The mercury thermometers used

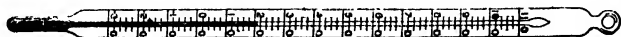


FIG. 33.—The Mercury Thermometer.

for practical work should be graduated in degrees, and should read from -10°C. to 110°C. In reading them each degree may be subdivided by eye into tenths, so that with care and a little practice such a thermometer will give readings to one-tenth of a degree. Owing to various sources of error, which cannot be discussed here, it is not worth while to attempt to read more accurately than this with a mercury thermometer unless numerous precautions and corrections are made.

EXPERIMENT 49.—To test the lower fixed point of a mercury thermometer.

Place a large funnel in the ring of a retort stand with a beaker below it to catch any liquid which may run through. Break some ice into small pieces, wash them in distilled water, and pour them into the funnel. Make a hole in the ice with a lead pencil, and insert the mercury thermometer in the hole until the 0° graduation is only just visible above the surface of the ice. A little distilled water may then be added to the ice in the funnel and the whole allowed to stand until the reading of the thermometer becomes constant. This may take five minutes. When the reading is found to remain constant over an interval of two minutes, read the thermometer carefully, estimating by eye to one-tenth of a degree. The position of the mercury in the stem is the true zero of the thermometer, and the difference between this and the scale reading gives the zero error. Record your results as follows:

Reading of thermometer in melting ice, $0^{\circ}\cdot3$.

Zero error = $-0\cdot3$.

The negative sign indicates that the zero marked on the

instrument is below the true zero, and that the correction must be subtracted from the actual reading of the instrument to obtain the real temperature.

The alcohol thermometer may be treated in the same way as the mercury thermometer; the position of the end of the column when in melting ice being marked with a scratch on the tube. This will be the zero point of the thermometer.

EXPERIMENT 50.—To test the upper fixed point of a mercury thermometer.

The temperature of boiling water is affected by the presence of dissolved impurities. Even when the water is pure the steam is often given off in violent series of spurts, in the intervals between which the temperature may rise considerably above the true boiling-point. This is known as "bumping." The upper fixed point is, therefore, always tested with the thermometer immersed not in the water, but in the steam given off, the temperature of which depends only on the barometric pressure. To ensure that the temperature of the thermometer shall reach that of the steam special double-walled chambers known as hypsometers are used. A simple hypsometer can easily be constructed from two pieces of glass tubing and two corks, as shown in Fig. 34. The inner tube A should be wide enough to allow the thermometer to hang inside it without its touching the side. A tube of about 1 cm. internal diameter will be convenient. The outer tube B may then be about 2 cm. diameter. The inner tube passes straight into the boiler C, which may be either a glass flask or, since glass vessels are fragile, a metal can, such as a small oil can. If a metal boiler is used, care should be taken to ensure that it does not boil dry during the experiment.

The thermometer is inserted through the cork in the wider tube so that it hangs inside the inner tube without touching it.

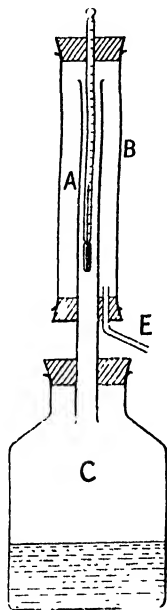


FIG. 34.—A Home-made Hypsometer.

The water in the boiler is boiled and steam passes up the inner tube, down the outer tube, and out by the escape-pipe E at the bottom. The thermometer is thus surrounded by a double jacket of steam.

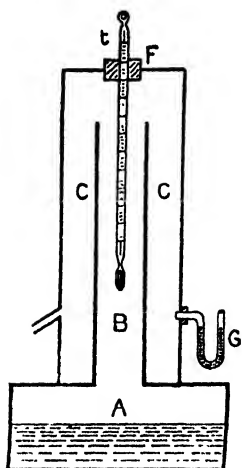


FIG. 35.—The Hypsometer.

When the column of mercury in the thermometer has become quite steady (which may take five minutes or more) the thermometer is adjusted if necessary so that the top of the column is just visible above the upper cork. The reading is then recorded, fractions of a degree being estimated by eye. The barometer is then read, and its height recorded.

The barometric height is required because the temperature of the steam varies with the atmospheric pressure. It is found that for pressures near the normal, an increase of pressure of 26.8 mm. of mercury increases the boiling-point by 1°C. The true boiling-point

of the thermometer can be calculated as in the following example :

Reading of the thermometer in steam . . . 100°.3
 Barometric height 752 mm.
 Difference between actual and normal atmospheric pressure
 = 760 - 752 mm. = 8 mm.

Actual temperature of steam

$$= 100^\circ - \frac{8}{26.8} = 100^\circ - 0.3 = 99^\circ.7$$

Error in boiling-point of thermometer

$$= 100^\circ.3 - 99^\circ.7 = 0^\circ.6$$

Boiling-point correction = -0°.6

If available, a proper hypsometer as illustrated in section in Fig. 35 may of course be substituted for the home-made apparatus described.

The errors in the boiling-point and freezing-point graduations of a thermometer are generally due to a gradual decrease in the volume of the bulb, a process which may go on for

many months after the bulb has been blown. If the stem has been correctly divided we should be able to determine the temperature corresponding to any graduation on the instrument from a knowledge of the errors in the fixed points. This, however, is not generally the case in practice, and for accurate work a table of corrections for the instrument is constructed by comparing its readings with those of a standard instrument, at different points on the scale. The method is indicated in the following experiment :

EXPERIMENT 51.—To compare the scales of a Centigrade and a Fahrenheit thermometer.

Clamp the two thermometers side by side by means of a wooden clamp so that their bulbs are as close together as possible. Place them in a vessel of water on a tripod stand so that the bulbs are well below the surface of the water but not in contact with the bottom or sides of the vessel. A small copper saucepan, or even an ordinary iron saucepan, makes a very convenient water bath and has the advantage that, unlike a glass beaker, it is not liable to be broken either by too sudden application of the flame, or by careless and over-vigorous stirring. A beaker, however, will serve if a metal vessel is not available.



FIG. 36.—A Convenient Stirrer.

The water in the bath is now stirred thoroughly, a very convenient form of stirrer being that shown in Fig. 36. It consists simply of a piece of fairly stiff brass or copper wire the lower end of which is bent into a circle of convenient diameter for the vessel in which it is to be used, the upper portion being at right angles to the plane of the circle and bent to form a handle. By moving this up and down in the liquid much more efficient mixing is produced than by stirring round and round with a rod. After stirring, the two thermometers are read, and their readings recorded.

The water in the bath is now heated until the Centigrade thermometer registers about 21°C. , and the flame is then taken away. The water is constantly stirred until the Centigrade thermometer falls exactly to the 20° graduation, and the Fahrenheit thermometer is immediately read. The process is repeated every 10° on the Centigrade thermometer up to the

boiling-point of water. By heating the water slightly above the temperature at which we wish to make our reading and allowing the temperature to fall gradually to the desired degree we ensure that the two thermometers shall have time to attain the temperature of the water, while the steady stirring ensures that the whole of the water bath shall be at the same temperature. In the absence of stirring, and especially while the water is being heated, very considerable differences of temperature may exist in different parts of the bath. If ice is available the thermometers may also be compared at temperatures below that of the room, by adding ice to the water. The reading at 10°C . should not be taken until all the ice has melted.

The results should be recorded in tabular form, and a graph should be drawn, the readings of the Centigrade thermometer being plotted along the horizontal axis, and the corresponding readings of the Fahrenheit thermometer on the vertical axis. The resulting graph should be a straight line, which, if both thermometers are correct, should cut the vertical axis at the 32°F . mark. The experimental points will probably not all lie exactly on a straight line. This may be due in part to slight unavoidable errors in reading the thermometers, and in part to actual errors in the thermometers themselves.

EXPERIMENT 52.—To graduate an unmarked thermometer.

The unmarked alcohol thermometer, constructed in Experiment 48, is clamped by the side of the Centigrade thermometer in place of the Fahrenheit thermometer used in the previous experiment, and the operations are repeated exactly in the same way, with the exception that when the mercury thermometer has fallen, say, to the 20°C . graduation, the position of the meniscus of the column in the alcohol thermometer is marked with a fine scratch by a sharp file. These scratches should be made for each 10° up to 60°C . The alcohol thermometer should not be heated above this temperature. The distances between successive scratches can be divided into ten equal parts to give single degrees, if their distance apart is sufficiently great to permit of this subdivision.

§ 18. DETERMINATION OF BOILING-POINTS AND MELTING-POINTS

A PURE liquid boils at a temperature which is characteristic of the liquid, and which is constant as long as the atmospheric pressure is constant. The boiling-point of a pure liquid can be determined accurately by use of the hypsometer.

EXPERIMENT 53.—To determine the boiling-point of a pure liquid.

The hypsometer shown in Fig. 34 may be used for the experiment. The inner tube is connected through a cork to a small flask containing the liquid whose boiling-point is required. Alcohol is a convenient liquid for the experiment. The exhaust tube *E* should be connected to a long glass tube, passing into a small flask immersed in a basin of cold water, in order to condense the vapour which passes over. In this way the liquid is recovered, and danger of explosion from the inflammable vapour coming into contact with the flame is removed. As a further precaution the flask containing the liquid should be heated in a water-bath. When the liquid is boiling freely, the thermometer inside the inner tube *A* is withdrawn, until the top of the mercury column is just visible above the cork. The reading of the thermometer, when it has become stationary, is the boiling-point of the liquid at the atmospheric pressure for the day. Record this temperature, and the corresponding barometric height.

The correction for change of boiling-point with change of atmospheric pressure differs for different liquids, and can only be determined by more elaborate experiments.

The presence of any dissolved solid in the liquid always raises the boiling-point, the amount of rise depending upon the nature of the dissolved substance, and the proportion of it present in the solution.

Since the temperature of the steam coming from a boiling solution of a salt in water will always be the same as that from

the pure liquid, the boiling-point of a solution must be determined by placing the thermometer in the liquid itself.

EXPERIMENT 54.—To determine how the boiling-point of a solution of salt in water varies with the quantity of salt which it contains.

Weigh out five quantities of common salt, each of 15 gm., and place them on separate pieces of paper on the bench. Measure out into a flask 200 c.c. of water, and place a few fragments of clean broken glass in the flask to prevent superheating. Heat the water over a bunsen flame, and when it begins to boil turn down the flame so that the liquid is just boiling. If the water is allowed to boil too rapidly a considerable amount of steam will escape, and the volume of water will become appreciably smaller. This could be entirely prevented by fitting the flask with a reflex condenser, but this will hardly be necessary for the present experiment.

Take the temperature of the boiling water by placing a thermometer, which must be graduated to read up to at least 110°C ., in the boiling liquid and noting carefully the steady reading. Then add one of the packets of salt, and when it has all dissolved and the liquid is again boiling, take the new temperature. Repeat the process until all the packets of salt have been added to the water.

Assuming that no appreciable quantity of water has boiled away, the concentration of the solution at any stage can be calculated from the weight of salt which has been added. For accurate work the thermometer used should be one whose boiling-point has been recently redetermined (Experiment 50). The boiling-point correction can then be added (or subtracted) from each of the readings. The results may be tabulated as follows :

Mass of water taken, 200 gm. Barometric height, 758 mm. Boiling-point correction of thermometer, $-0^{\circ}6$.			
Mass of Salt added, in gm.	Composition of Solution in gm. per 100 gm. of Water.	Thermometer Reading.	Temperature (Corrected).
15	7.5	$101^{\circ}7$	$101^{\circ}1$
30	15.0	$103^{\circ}1$	$102^{\circ}5$
45	22.5	$104^{\circ}9$	$104^{\circ}3$
60	30.0	$106^{\circ}8$	$106^{\circ}2$
75	37.5	$108^{\circ}0$	$107^{\circ}4$
80.	40.0	$109^{\circ}6$	$109^{\circ}0$

These results should be plotted on squared paper, the composition of the solution (second column of the table) being plotted along the horizontal axis, and the corresponding boiling-point (final column) along the vertical axis.

Some solids have a perfectly definite melting-point, the change from solid to liquid or from liquid to solid taking place at a perfectly definite temperature, which remains constant until the whole of the substance has been changed from one state to the other. Ice is a good example of this class, hence its value for determining the lower fixed point of a thermometer. Other solids, however, pass through a transition stage, the substance generally becoming plastic or pasty, and the temperature rising gradually all the time. Such solids have no definite melting-point.

EXPERIMENT 55.—To determine the melting-point of a solid.

Draw out a piece of glass tubing into a thin-walled capillary tube, and seal off the lower end. Take a few fragments of the solid and insert them into the capillary tube, and fasten the tube to the bulb of a thermometer by means of rubber bands (Fig. 37). Heat some water slowly in a beaker, and place the thermometer and tube in the water, stirring all the time with the thermometer. Watch the contents of the tube closely, and at the instant when it is seen to liquefy read the temperature of the thermometer. The change of state is generally quite easy to observe, as the solid fragments will usually be opaque, while the liquid will be transparent.

Heat the water 1 or 2° higher, and then allow to cool, still stirring gently all the time. Note the reading of the thermometer when the substance solidifies. Repeat the operations three or four times until you are sure that the readings are satisfactory. The mean of the temperatures at which the solid liquefies, and the liquid solidifies, is taken as the melting-point of the solid. Determine in this way the melting-points of naphthalene, beeswax, and stearic acid.



FIG. 37.—
Determination of
a Melting-
Point.

EXPERIMENT 56.—To determine the melting-point of a solid by plotting a cooling curve.

Much information as to the process of solidification can be obtained from experiments on the rate of cooling of the substance as it passes through its change of state. Place sufficient naphthalene in a large test-tube almost to fill it, and heat gently in a bunsen flame until the whole is liquid. If necessary add more naphthalene until the tube is about two-thirds full. Insert a mercury thermometer in the liquid and heat to a temperature of about 95°C . Then clamp the tube in a wooden clamp, the thermometer being held in a second clamp

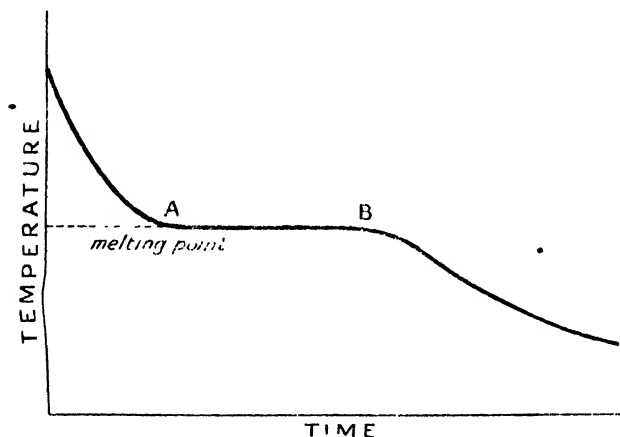


FIG. 38.—Cooling Curve for a Substance undergoing Solidification.

so that it does not touch the sides of the test-tube, and allow the whole to cool.

Read the temperature every half-minute, recording both the temperature and the time. Continue the readings until the whole of the substance has solidified and is well below its melting-point, say down to a temperature of 60°C . Plot the results on squared paper, as shown in Fig. 38. The resulting graph is known as a cooling curve.

It will be found that there is one stage, such as AB, in the cooling when the temperature remains practically constant and the curve becomes parallel to the axis of time. This temperature is the melting-point of the substance. As the substance

is obviously giving out heat during the whole of the cooling process, the fact that the temperature at one stage remains constant must be due to the emission of heat by the substance itself during the change of state. Thus during solidification the substance gives out heat. This is known as the latent heat of solidification.

In the case of substances which have an indefinite melting-point, the cooling curve becomes very much flattened as the substance passes through its transition point, owing to the emission of latent heat during the process, but never becomes quite horizontal. Repeat the above experiment with paraffin wax instead of naphthalene, and note the difference in the shapes of the two cooling curves.

§ 19. THE MEASUREMENT OF EXPANSION OF SOLIDS AND LIQUIDS

THE EXPANSION OF SOLIDS

SOLIDS increase in length when heated, but the effect is generally very small. For example, a copper bar 1 metre long at 0°C . would increase its length by rather less than 2 millimetres if heated to 100°C . Obviously very accurate measuring apparatus will be required to measure the expansion of a solid with any accuracy, if the measurement is to be made directly. A screw gauge or a spherometer will measure correctly to at least $\frac{1}{1000}$ th of a millimetre and will thus give us an accuracy of at least 1 per cent., if it can be applied to the problem. The spherometer can be used for measuring the expansion of a solid in the following way :

EXPERIMENT 57.—To measure the expansion of a metal by means of a spherometer.

A brass tube about half a metre long is closed at both ends, but is provided with side tubes BB near each end (Fig. 39). It is supported in a vertical position by a stand S, with the lower end resting on a sheet of plate glass G. A second glass plate having a hole in the centre is supported on a stout retort ring so that its lower surface is about $\frac{1}{2}$ cm. above the top of the tube, and an ordinary spherometer is placed on the plate with its middle leg projecting through the hole. By means of a rubber tube attached to the upper tube B pour some water (which has been allowed to stand for some time so as to attain the temperature of the laboratory) through the tube, allowing it to flow into a beaker by the lower tube. Take the temperature of the water after passing through the tube. This will give the temperature of the tube. Without unnecessary delay screw down the centre leg of the spherometer until it just touches the upper end of the tube A, and take the reading (see Experiment 5).

Then screw up the leg until there is ample clearance between it and the tube.

In the meantime, boil some water in a flask or a metal boiler fitted with a cork and delivery tube, and pass the steam through the tube A by connecting the delivery tube to the upper side tube B. Allow the steam to pass freely for five minutes, and then again screw down the middle leg of the spherometer until it is just touching the upper end of the tube A, and take the reading. Continue to pass the steam for another two minutes and again make the spherometer reading. (Do not leave the spherometer in contact with the tube between the two readings.) If the two readings are the same, the tube has reached its maximum expansion for the temperature of the steam. Stop the supply of steam and pour cold water through the tube until it has again reached the temperature of the room. Adjust the spherometer again and take the reading. This should be the same as the first reading. The difference between the readings of the spherometer with the tube hot and cold gives the expansion of the tube for a rise in temperature equal to the difference between the temperature of steam and the temperature of the room.

The tube may now be dismantled and its length at room temperature measured. As the tube is about 50 cm. long this can be done with sufficient accuracy by laying it along a metre scale. The barometer is then read, and the temperature of the steam is deduced from it by the method explained in Experiment 50. Unless the pressure happens to be considerably removed from the normal the correction will barely be appreciable in comparison with the other errors of experiment, but it affords good practice. The coefficient of expansion can then be worked out as in the following record of an experiment :

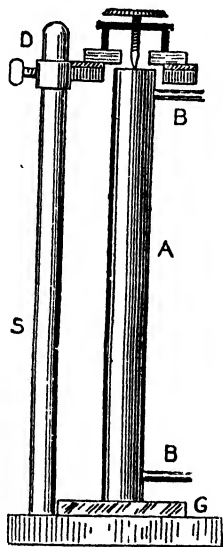


FIG. 39.—Apparatus for the Measurement of the Linear Expansion of a Metal Tube.

Length of tube at the temperature of the room, 56.62 cm.

Spherometer reading (tube cold), . . . 5.235 mm.

„ „ (steam passing through
tube), . . . 4.360 „

„ „ (tube cold), . . . 5.235 „

Expansion of tube, 0.875 mm. = 0.0875 cm.

Initial temperature of tube, . . . 14.7 C.

Barometric height, . . . 756 mm.

Temperature of steam = 100° C. - $\frac{4}{26.8}^{\circ}$ = 99.8 C.

Rise in temperature of tube = 99.8 - 14.7 = 85.1.

Coefficient of expansion of tube

$$= \frac{\text{increase in length of tube}}{\text{length of tube} \times \text{rise in temperature}}$$

$$= \frac{0.0875}{56.62 \times 81.1} = 0.0000181 \text{ per } ^{\circ} \text{C.}$$

Owing to loss of heat by radiation and convection, the walls of the tube A are probably slightly below the temperature of the steam. This can be prevented by "lagging" the tube, *e.g.* by winding one or two layers of string round it, but the error is usually small. If desired, the expansion of a solid rod can be measured in a similar way by jacketing the rod with a wider tube through which water and steam can be passed in turn. The rod is arranged so that its two ends just project through corks which close the ends of the jacket, and the experiment proceeds as already described.

An alternative method to the one described is to magnify the small expansion in some definite known ratio, generally by the application of the principle of the lever. If a lever ACB is pivoted at C, and the end A moves a small distance, say to A', then by similar triangles

$$\frac{AA'}{AC} = \frac{BB'}{BC} \text{ or } BB' = \frac{AA' \cdot BC}{AC}$$

Thus, if the ratio $\frac{BC}{AC}$ is large, a small motion of the end A will produce a large movement of the end B. The distance AC must not be too small to be measured accurately, and the lever

must be rigid enough not to bend during the experiment. These conditions are somewhat difficult to realise mechanically.

THE EXPANSION OF LIQUIDS

All liquids expand with rise of temperature, though, as we shall see later, water over a certain range of temperature is an exception. Liquids, however, require to be contained in some sort of vessel. The expansion actually observed in any experiment is, therefore, the difference between the expansion of the liquid and the expansion of the vessel. This is known as the *relative* or *apparent* expansion of the liquid. The actual increase in volume of the liquid itself is called the *absolute* expansion of the liquid. The coefficient of cubical expansion of liquids is much greater than that of solids, but is still quite small. Thus, a mass of mercury which occupied 100 c.c. at 0° C. would occupy about 101.8 c.c. at 100° C. The accuracy of our experiments will be greatly increased if we can arrange to make our measurements depend not directly on the volume but on the weight of the liquid, since, with a good physical balance, weight can be measured more accurately than any other quantity. Thus, 1.8 c.c. of mercury would weigh rather more than 24 gm., a weight which could easily be determined to an accuracy of 1 part in 2000. The experiment can easily be performed with an ordinary specific gravity bottle (see Experiment 16).

EXPERIMENT 58.—To determine the coefficient of apparent expansion of a liquid by the specific gravity bottle.

It is proved in text-books of physics that if W_1 is the weight of a liquid which completely fills a given vessel at a temperature T_1 , and W_2 the weight which completely fills the same vessel at a temperature T_2 , the coefficient of apparent expansion of the liquid is equal to

$$\frac{\text{loss of weight on heating}}{\text{weight of hot liquid} \times \text{difference in temp.}}, \text{ i.e. to } \frac{W_1 - W_2}{W_2 (T_2 - T_1)}$$

Clean and dry a specific gravity bottle, and weigh it carefully. Fill it with the liquid whose coefficient of expansion is to be determined (glycerine or olive oil are suitable liquids for the purpose). Insert the stopper, and remove with blotting-paper all the liquid which overflows through the hole in the stopper. Take the temperature of the liquid used for filling the bottle.

Transfer the bottle to the balance pan, handling the bottle as little as possible and preferably with a clean duster, to avoid warming the liquid, and weigh again. Now suspend the bottle by a thread in a beaker or pan of boiling water, so that the bottle is completely immersed up to the neck, and allow it to remain for at least ten minutes, with the water boiling gently all the time. A liquid contained in a small bottle, such as a specific gravity bottle, takes a considerable time to attain the temperature of its surroundings, especially if it is viscous like glycerine, and considerable error will arise if the bottle is removed too soon from the bath. The temperature of the boiling water may be taken with a thermometer, for which the boiling-point correction is known. If the boiling-point correction of the thermometer is not known it will probably be more accurate to calculate the boiling-point of water from the barometric height.

The bottle is then removed from the bath, carefully dried, and the thread used for suspending it removed. When sufficiently cool the bottle is again weighed. The coefficient of apparent expansion of the liquid can be calculated as in the following record :

Weight of bottle, empty $\quad \quad \quad = 20.18 \text{ gm.}$

Weight of bottle full of glycerine, before heating $= 83.18 \text{ „}$

Weight of bottle and glycerine, after heating $= 80.47 \text{ „}$

Initial temperature $\quad \quad \quad = 16^{\circ}.2 \text{ C.}$

Temperature of bath $= 99^{\circ}.6 \quad \therefore \text{ Rise in temp.} = 83^{\circ}.4$

Loss of weight on heating $= 83.18 - 80.47 = 2.71 \text{ gm.}$

Weight of glycerine remaining in bottle, after heating
 $= 80.47 - 20.18 = 60.29 \text{ gm.}$

Coefficient of apparent expansion of glycerine

$$= \frac{2.71}{60.29 \times 83.4} = 0.000541 \text{ per } ^{\circ} \text{C.}$$

The experiment is sometimes carried out with a weight thermometer in place of the specific gravity bottle. The weight thermometer (Fig. 40) consists of a fairly wide glass tube closed at one end, the other end being drawn out into a fine capillary tube. The weight thermometer is weighed empty and is then filled by placing the end of the capillary tube beneath the surface of the liquid contained in a small beaker, and then heating and cooling the bulb in the way described in Experiment 48. When quite full, the bulb of the weight

thermometer is placed in a beaker of water at room temperature with the capillary tube still dipping into the liquid in the beaker, and allowed to remain for about ten minutes. The capillary tube is then withdrawn from the liquid, and the thermometer is full at the temperature of the bath, which is taken with a thermometer. The end of the capillary is dried with blotting-paper. As the bath will by now probably be one or two degrees hotter than the room temperature, there is no fear of liquid being forced out of the weight thermometer during weighing. The experiment then proceeds exactly as with the density bottle.

The advantage of the weight thermometer over the specific gravity bottle lies in the fact that the stopper of the specific gravity bottle may be slightly dislodged by the expanding liquid, thus altering slightly the volume of the bottle. The weight thermometer is, therefore, more reliable, but it takes much longer to fill.

It can be shown that the coefficient of absolute expansion of the liquid is approximately equal to the sum of the coefficient of apparent expansion of the liquid as measured by the weight thermometer and the coefficient of cubical expansion of the containing vessel. The coefficient of linear expansion of ordinary glass is about 0.000008 , and its coefficient of cubical expansion is, therefore, about 0.000024 . If this is added to the coefficient of apparent expansion obtained in the previous experiment it will give the coefficient of absolute expansion of the liquid. The coefficient of absolute expansion of glycerine as determined from the measurements in the example is thus $0.000541 + 0.000024 = 0.000565$ per degree Centigrade.

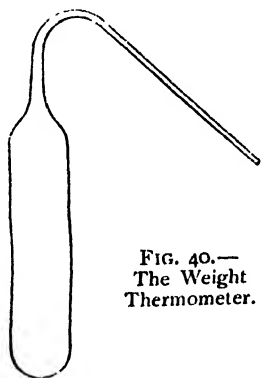


FIG. 40.—
The Weight
Thermometer.

The experiment with the weight thermometer or gravity bottle obviously measures the *average* expansion of the liquid between the two temperatures employed. Many liquids, however, do not expand uniformly, that is to say, their expansion for a rise of 1° C. is different in different parts of the

scale of temperature as measured by a mercury or an air thermometer. Water is an extreme example, as between temperatures of 0°C . and 4°C . its volume actually diminishes as the temperature rises. The effect can be studied as in the following experiment :

EXPERIMENT 59.—To study the change in volume of water with change in temperature.

Procure a small flask of about 100 c.c. capacity and a well-fitting rubber cork bored with a single hole, into which there fits a length of about 18 cm. of capillary tubing of about 1 mm. internal diameter (Fig. 41*a*). Measure the volume of the flask, and place in it a volume of mercury equal to one-seventh the volume of the flask. For a given change of temperature mercury expands seven times as much as an equal volume of glass. The increase in volume of the mercury will thus exactly balance the increase in volume of the flask, and the volume of the space above the mercury will be the same at all temperatures. Any change observed in the level of the water must, therefore, be due to its own expansion or contraction.

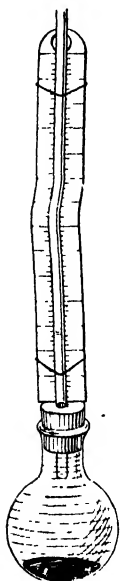


FIG. 41*a*. — Apparatus for investigating the Expansion of Water at Different Temperatures.

Fill the flask to the brim with distilled water which has been recently boiled, and then press home the cork. This will force the water up the capillary tube. The water should rise about four or five centimetres up the stem, and there should, of course, be no air left in the bulb or tube. Fasten a steel scale to the capillary tube by rubber bands, and place the bulb in a water bath containing water at room temperature, so that the bulb is completely immersed. When the bulb has acquired the temperature of the bath, that is, when the level in the tube remains steady, read the level of the top of the water column in the capillary tube and the temperature of the bath.

Cool the bath by dropping small pieces of ice into it, stirring the bath constantly, and when the thermometer has fallen two degrees read again the level of the water column.

Continue the process, making readings of the water column every two degrees down to 0°C .

Represent your results by a graph plotting the temperature along the horizontal axis, and the corresponding levels of the water column along the vertical axis. It will be seen that for temperatures above 4°C . water contracts when cooled, like other liquids; below this temperature water expands on cooling. The curve is shown in Fig. 41*b*.

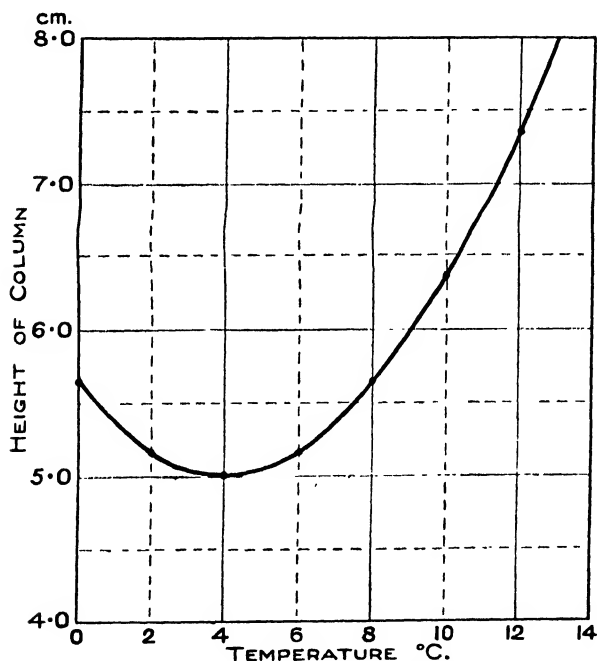


FIG. 41*b*.—The Expansion of Water.

The experiment can be made to give quantitative results if the volume of water used is measured and the area of cross-section of the tube. The former can be obtained by weighing the flask and mercury together with the cork and tube before filling with water, and again with the water in the apparatus. The difference between the two weighings gives the weight of water used, and hence the volume. The area of cross-section of the tube is found before the experiment by finding the weight

of mercury necessary to fill it. (Experiment 14.) Let L be the reading of the top of the water column at 4°C . The volume of water at this temperature is equal to its mass in grams, by definition ($=V$ c.c.). Let L_1 be the reading at some other temperature T_1 . The volume at this temperature is then obviously $V + A(L_1 - L)$, where A is the area of cross-section of the tube. The expansion between any two temperatures T_2 and T_1 is similarly equal to $A(L_2 - L_1)$, and the mean coefficient of expansion of water between these two temperatures is

$$\frac{(L_2 - L_1) A}{(V + L_1 A)(T_2 - T_1)}.$$

The mean coefficient of expansion of water between any two temperatures included in the graph can thus be determined. Calculate in this way the coefficient of expansion of water between the temperatures of 0° and 4° , 0° and 15° , 4° and 15° . Note that the coefficient of expansion between 0° and 4° is negative.

The experiment can also be performed using a capillary tube with a bulb blown on one end as in Experiment 48.

EXPERIMENT 60.—To measure the mean coefficient of apparent expansion of water between different temperatures by the water thermometer.

A capillary tube with an internal diameter of cross-section of 0.1 mm. and a bulb of about 2 cm. diameter will give a sufficiently open scale for the experiment. Dry the tube and bulb if necessary, and weigh it. Warm the bulb very gently (the warmth of the hand will generally be sufficient), and place the open end under mercury, holding the tube as nearly horizontally as possible. When cold withdraw the tube from the mercury, and measure the length of the mercury column. Expel the mercury into a weighed crucible or watch-glass by gently warming the bulb, and find the weight of mercury. Calculate the average area of cross-section of the tube.

Fill the apparatus with distilled water by the method of Experiment 48, so that the water fills the tube at about 70°C . Allow it to cool, and re-weigh, thus determining the mass of water in the apparatus. Fasten a steel scale to the tube by rubber bands to enable the position of the top of the column to be read.

Now place the bulb in a water bath and cool down to 0°C .

by adding ice. Read the level of the top of the water column. Heat the bath slowly, taking the temperature and level every 2° up to 10° C., and after that every 5° up to 60° C. Plot the results as described in the previous experiment. Calculate, by the method already described, the coefficient of expansion between temperatures of, say, 0° and 15° , 0° and 30° , 30° and 60° . Since there is no mercury in the bulb these will be the coefficients of apparent expansion of water between the various temperatures.

The expansion of other liquids can be investigated in the same way.

EXPERIMENT 61.—To determine the coefficient of absolute expansion of a liquid.

The coefficient of absolute expansion of a liquid can be determined by the following simple modification of Dulong and Petit's apparatus.

A tall U-tube (Fig. 42) is mounted with its limbs A, B vertical. To prevent diffusion between the two limbs, it is preferable to have the horizontal tube C made of capillary tubing, but this is not absolutely essential; an ordinary U-tube will serve. Each limb is jacketed with a wide glass tube, one of the jackets, A, being furnished with corks top and bottom as shown in the figure, so that steam can be passed through it. The other jacket, B, is filled with cold water, or preferably with broken ice. The liquid to be experimented on is poured into the U-tube until it stands just above the level of the top of

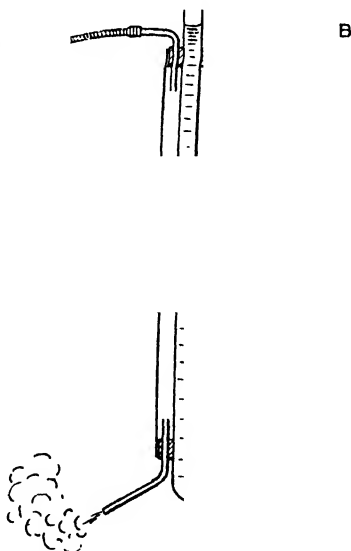


FIG. 42.—Determination of the Coefficient of Absolute Expansion. Dulong and Petit's Method.

the U-tube until it stands just above the level of the top of

the jackets, and steam is passed through the steam jacket for about five minutes. The height of the hot and cold columns of liquid above the middle point of the capillary tube C are measured as described in Experiment 22. Let them be h_{100} and h_0 respectively. Then, if ρ_{100} is the density of the liquid at 100°C. , and ρ_0 that at 0°C. , we have by the principle of the U-tube (Experiment 22):

$$h_0 \rho_0 = h_{100} \rho_{100}$$

$$\frac{\rho_0}{\rho_{100}} = \frac{h_{100}}{h_0}$$

But

$$\rho_0 = \rho_{100} (1 + 100\alpha)$$

where α is the coefficient of absolute expansion of the liquid.

Hence

$$\alpha = \frac{h_{100} - h_0}{100 h_0}$$

If the temperatures of the two limbs are T and t respectively instead of 100° and 0° , we have similarly

$$\alpha = \frac{h_T - h_t}{(T - t) h_t}$$

This experiment is not capable of much accuracy in its simple form, partly owing to difficulties in measuring the lengths of the columns, partly owing to uncertainties as to the exact temperature of the exposed parts of the tube. Aniline is a good liquid to use, as its coefficient of expansion is high, and the difference $h_{100} - h_0$ is fairly large.

§ 20. THE EXPANSION OF GASES

GASES are far more expansible than liquids. A mass of air which has a volume of 100 c.c. at 0°C . will have a volume of 136.6 c.c. at a temperature of 100°C . if the pressure remains the same. Since gases must be contained in some vessel, our experiments really measure the coefficient of relative expansion of the gas, but on account of the large expansion of the gas the correction for the expansion of the containing vessel is so small (less than 1 per cent. in the case of a glass vessel) that it can be neglected, except for very accurate work. Since, however, the volume of gas varies with the pressure as well as with the temperature, it will be necessary to specify the pressure conditions during the experiment.

EXPERIMENT 62.—To measure the coefficient of expansion of air at constant pressure. (First method.)

Take a piece of thick-walled capillary glass tubing, having a bore of internal diameter rather less than 1 mm. Seal off one end of the tube, taking care to interfere as little as possible with the cross-section of the bore at the sealed end. Heat the tube carefully, and dip the open end under the surface of a little mercury until a column of mercury about 2 cm. long has entered the tube. The end is then removed from the mercury. When the tube has cooled to the temperature of the laboratory, the mercury thread, which now encloses a definite mass of air in the tube, should be about half-way along the tube. The position of the mercury index depends on the temperature to which the tube was heated, and this must be adjusted by trial until the index comes in the right position.

Place the tube in a vertical position, open end upwards, in a tall metal can or small gas jar, and surround it with small lumps of ice. Pour a little water on the ice, and when the tube has had time to reach the temperature of the ice, withdraw the tube sufficiently for the index to be visible, and make a mark with a sharp file against the position of the lower end

the corresponding values for 100° C. Join the two points so obtained by a straight line. Since air is the standard thermometric substance, its expansion is uniform *by definition*, and the length of the air column at any temperature will thus be represented by the corresponding point on the graph. Produce the straight line to 0° C., and read off the corresponding length of air column. Calculate the coefficient of expansion as in the previous example, using the value thus obtained. This procedure is necessary, as the expansion of a gas is always measured from 0° C.

The experiment just described, though instructive, is not very accurate. The mercury index is apt to stick. This error may be reduced by tapping the tube gently before taking a reading. The mercury index also not infrequently fails to act as an effective seal and a little of the air may escape past the index. A more accurate method is indicated in the following experiment :

EXPERIMENT 63.—*To determine the coefficient of expansion of air at constant pressure. (Second method.)*

A round-bottomed 250 c.c. flask (Fig. 43) is closed with a well-fitting rubber cork bored with a single hole through which passes a short length of glass tubing, the outer end of which is closed with a short piece of rubber tubing and a clip. The apparatus, which is sometimes known as a constant pressure air thermometer, is dried and weighed. It is then placed in a bath of boiling water, the clip of course being opened, and after leaving it for some minutes to attain the temperature of the bath, the clip is closed securely.

The flask is then completely immersed mouth downwards in a large basin of water, and the clip is opened. Water rushes in to a certain level to replace the air which has been expelled at the higher temperature. Lower or raise the flask until the level of the water is the same inside and out. The pressure of the gas in the flask will then be atmospheric, while its volume is obviously the volume of gas which com-



FIG. 43.—Apparatus for determining the Expansion of Air at Constant Pressure.

pletely filled the flask at 100°C . Close the clip, dry the outside of the flask, and weigh. Take the temperature of the bath. Fill the flask and tube completely with water and weigh again. The coefficient of expansion of the air at constant pressure may be calculated as in the following example :

Weight of flask, etc., empty = 92.2 gm.

Weight of flask and water after

opening clip under water = 146.8 „

Temperature of water in bath = 19.6

Weight of flask full of water = 344.4 gm.

Volume of flask = $344.4 - 92.2 = 252.2$ c.c.

Volume of air in flask after

opening clip under water = $344.4 - 146.8 = 197.6$ c.c.

$$\frac{\text{Volume of air at } 100^{\circ}}{\text{Volume of air at } 19.6^{\circ}} = \frac{252.2}{197.6} = \frac{1 + 100\alpha}{1 + 19.6\alpha}$$

where α is the coefficient of expansion of the air.

Hence $\alpha = 0.00368$ per $^{\circ}\text{C}$.

If the volume of a given mass of gas is kept constant the pressure increases with rise in temperature, the relation between pressure and temperature being given by the equation

$$p_t = p_0 (1 + \alpha t)$$

where p_0 and p_t are the pressures at 0°C . and $t^{\circ}\text{C}$. respectively, and α is the coefficient of increase in pressure with temperature. The value of α can be determined by means of constant volume air thermometer. For a perfect gas it is numerically equal to the coefficient of expansion of the gas at constant pressure.

EXPERIMENT 64.—To determine the relation between the temperature and pressure of a gas at constant volume.

A convenient form of constant volume air thermometer is shown in Fig. 44. It consists of a bulb, A, about 6 cm. in diameter, blown at one end of a thick-walled capillary tube, B, of about 1 mm. bore. The end of this capillary tube is joined by a long piece of rubber pressure tubing to a second glass tube, D, which is carried by a slide so that it moves up or down a vertical scale. A mark, M, is made on the capillary tube B, and the level of the mercury in the tube is to be kept at the mark M throughout the experiment. The volume of gas in the apparatus is thus kept constant.

The bulb A is completely immersed in broken ice in a metal pan or beaker, and while it is attaining the temperature of the ice the barometer is read. The mercury in the tube B is adjusted by raising or lowering the tube D until it stands level with the mark M. The level of the mercury in D is then measured on the vertical scale. The pressure on the air in A is then equal to the barometric height plus the difference in levels of the two mercury columns, if the mercury in D is above the mercury in B, or to the barometric height less the difference in level if the mercury in D is below that in B.

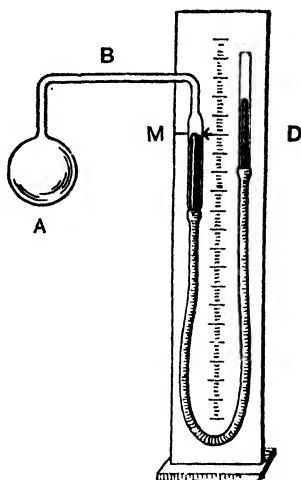


FIG. 44.—Constant Volume Air Thermometer.

The bulb A is removed from the ice and placed in a bath of boiling water. The air expands and D will have to be raised to keep the mercury in B at the level M. When the columns are stationary, adjust the mercury level to the mark M accurately, and read the level of the mercury in D on the scale. The coefficient of increase of pressure with temperature can be calculated as follows:

Barometric height, 752.4 mm. = 75.24 cm.

Reading of mark M on scale, 30.00 "

Reading of mercury in D at 0° C., 26.45 "

" " " at 100° C., 53.01 "

Difference in level of mercury columns at 0° C.

$$= 26.45 - 30.00 = -3.55 \text{ cm.}$$

Difference in level of mercury columns at 100° C.

$$= 53.01 - 30.00 = +23.01 \text{ cm.}$$

Pressure at 0° C. = 75.24 - 3.55 = 71.69 cm.

Pressure at 100° C. = 75.24 + 23.01 = 98.25 cm.

Increase in pressure for a

$$\text{rise of } 100^\circ \text{ C.} = 98.25 - 71.69 = 26.56$$

Coefficient of increase of pressure with temperature

$$= \frac{\text{Increase in pressure}}{\text{pressure at } 0^\circ \times \text{rise in temperature}} = \frac{26.56}{71.69 \times 100} \\ = 0.00370 \text{ per } ^\circ \text{ C.}$$

EXPERIMENT 65.—To determine the boiling-point of a saturated solution of common salt by the constant volume air thermometer.

The constant volume air thermometer can now be used to measure temperature, e.g. the boiling-point of a saturated solution of common salt. Determine the pressure of the air in the thermometer, exactly as in the previous experiment, in melting ice and in boiling water, and plot the two sets of readings on graph paper. Join the two points by a straight line and produce this line in both directions. The line so obtained gives the relation between pressure and temperature for the gas in the given instrument.

Now immerse the bulb of the instrument in the saturated salt solution and bring it to the boil. When the mercury columns have become stationary, adjust the mercury to the mark M, read the level of the mercury column in D, and calculate the pressure of the gas, as before, taking care not to neglect adding on the barometric height. Look up the corresponding pressure on the graph and read off the corresponding temperature. This will be the boiling-point of the salt solution.

It is most important that in determining temperatures by the constant volume air thermometer both the boiling-point and freezing-point should be previously determined, and the graph for the instrument drawn. Perfect gases increase in pressure by $\frac{1}{273}$ rd of their pressure at 0°C. for each degree rise in temperature. The gas in the bulb of the thermometer, however, owing generally to its not having been perfectly dried, will probably have a coefficient differing appreciably from the theoretical value, and errors amounting to as much as 10° or more may be made if only one of the fixed points, say, the freezing-point, is taken, and the theoretical coefficient employed.

The temperature of the boiling salt solution can be obtained from the measurements by calculation without drawing a graph as follows :

$$\begin{aligned} \text{Pressure of air at } 0^{\circ}\text{C.} &= P_0 \\ \text{,, ,, } 100^{\circ}\text{C.} &= P_{100} \\ \text{,, ,, } T^{\circ}\text{C.} &= P_T \end{aligned}$$

$$\alpha = \frac{P_{100} - P_0}{100P_0}$$

$$\text{Also } P_T = P_0(1 + \alpha T)$$

$$\therefore T = \frac{P_T - P_0}{P_{100} - P_0} 100$$

§ 21. CALORIMETRY

MEASUREMENT OF QUANTITIES OF HEAT

THE unit quantity of heat is the heat required to raise the temperature of 1 gm. of water through an interval of temperature of 1° C. It is known as the *calorie*. † Very careful experiments have shown that the quantity of heat required to raise the temperature of 1 gm. of water through 1° differs slightly at different temperatures, but the difference is so small that it is quite inappreciable from our point of view.

The apparatus used for measuring quantities of heat is called a calorimeter. The usual form of calorimeter employed for elementary experiments consists of a small metal vessel of cylindrical shape which is generally made of copper or aluminium. It is important that as little heat as possible shall escape from the calorimeter during the experiments. The surface of the calorimeter should, therefore, be brightly polished to reduce loss of heat by radiation, and the calorimeter itself should be surrounded by a larger metal vessel which is brightly polished on the inside. This still further reduces radiation losses, and has the additional advantage that it prevents thoughtless handling of the calorimeter itself, which is a very fruitful source of error if an exposed calorimeter is employed.

Conduction of heat from the calorimeter is reduced by standing the inner vessel upon some badly conducting substance, such as a *dry* cork, or by suspending it by strings from the outer vessel. In order to reduce the loss of heat by convection currents in the air, the space between the two vessels is sometimes filled with cotton-wool, and the outer calorimeter is closed with a lid (Fig. 45). The value of the cotton-wool is somewhat doubtful, but if it is employed care must be taken that the wool does not become damp through liquid splashing upon it from the inner vessel. With these precautions the rate of loss of heat from the inner vessel is very small, and,

if the experiments are performed smartly, the error introduced from this source will be inappreciable.

If a weighed quantity of water is placed in the calorimeter, the amount of heat given to the water during an experiment is equal to the mass of water multiplied by the rise in temperature. The calorimeter itself, however, also rises in temperature during the experiment, and this requires a certain quantity of heat

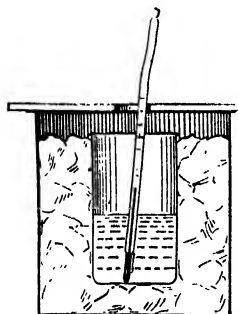


FIG. 45.—A Calorimeter.

which must be estimated in some way, and allowed for in the calculations. The number of calories of heat required to raise the temperature of the calorimeter 1° is known as the *water equivalent* of the calorimeter. If the specific heat of the metal of the calorimeter is known, the water equivalent can be calculated by multiplying the mass of the calorimeter by its specific heat. This assumes that the whole of the calorimeter becomes heated to the temperature of its contents, and it is for this

reason that the calorimeter is made of a good conducting substance such as copper. If a glass vessel were employed the amount of heat absorbed by the glass would be uncertain, and would probably differ in different experiments. The water equivalent of a calorimeter can also be determined by direct experiment as follows :

EXPERIMENT 66.—To determine the water equivalent of a calorimeter.

Weigh the calorimeter, and place in it sufficient water to about one-quarter fill it, and weigh again. Place the calorimeter in its protecting case and take the temperature as accurately as possible with a thermometer. In the meantime boil some water in another vessel, and when the water is boiling pour into the calorimeter a quantity of the boiling water of approximately the same volume as that already in it, so that the calorimeter will now be about half full. This should be done quickly, so as to avoid loss of heat from the boiling water, but care must be taken at the same time to avoid the liquid splashing out of the calorimeter. Stir the mixture with a stirrer of the shape illustrated in Fig. 36, and

when the temperature has reached its highest point, at which it will remain stationary for a short time, record it. The calorimeter and its contents are then weighed again to ascertain the mass of water added. The calculation of the water equivalent can then be made as follows :

Mass of calorimeter and stirrer	88.5 gm.
" " stirrer, and water	129.7 "

∴ Mass of cold water = 41.2 gm.

Initial temperature of calorimeter and water	15.6° C.
--	----------

" " hot water	100° C.
---	---------

Temperature of mixture	51.2° C.
----------------------------------	----------

Mass of calorimeter and contents after mixing	166.4 gm.
---	-----------

∴ Mass of hot water added = 166.4 - 129.7 = 36.5 gm.

Heat gained by calorimeter and water = heat lost by hot water

$$41.2(51.2 - 15.6) + W(51.2 - 15.6) = 36.5(100 - 51.2)$$

∴ W = water equivalent of calorimeter (and stirrer) = 8.8 gm.

The water equivalent as thus determined will include that of the thermometer, which is, however, very small, and is usually neglected.

DETERMINATION OF SPECIFIC HEATS BY THE METHOD OF MIXTURES

If a mass M_1 of the substance at a temperature t_1 is added to a mass M_2 of water contained in a calorimeter of *water equivalent* W and at a temperature t_2 , the mixture will attain an intermediate temperature t_3 . If s is the specific heat of the substance, we have

Heat gained by calorimeter and water = heat lost by substance

$$(M_2 + W)(t_3 - t_2) = M_1s(t_1 - t_3)$$

from which s , the only unknown in the equation, can be determined.

EXPERIMENT 67.—To determine the specific heat of a solid.

Certain preliminary precautions must be taken to secure an accurate result. In the first place, it must be remembered that the specific heat of most solids is very small compared

with that of water. Thus, if the mass of the solid used for the experiment is small compared with that of the water in the calorimeter, the heat given to the water by the solid will only raise its temperature very slightly, perhaps not more than a degree or so. This interval of temperature is far too small to be measured accurately with the thermometers generally employed, and the experiment is doomed to failure from the start. A rise in temperature of at least 10° is required if it is to be measurable with an accuracy of 1 per cent. This can only be assured by taking a sufficiently large mass of the solid. The mass of solid used should not be less than that of the water in the calorimeter, and it will often be an advantage to have it even greater. The attempt to carry out the experiment with only a small quantity of the solid is one of the most usual sources of failure in these experiments.

If a large mass of solid is taken, however, it should be broken into small fragments. Otherwise it will take a considerable time for the solid to attain the temperature of the water in the calorimeter. As the latter will be losing heat all the time, an appreciable error may be introduced from this cause. The experiment may be carried out as follows :

Place a quantity of the solid (say a quantity of iron tacks) in a boiling tube, and place a thermometer in the tube so that its bulb is surrounded by the substance. Place the boiling tube in a beaker or pan of water and boil for about ten minutes. A little cotton-wool in the mouth of the boiling tube will keep the steam from entering and condensing on the substance. While the water is boiling, weigh the calorimeter, add sufficient water to fill about one-third of the calorimeter and weigh again, thus obtaining the weight of water. By this time the thermometer in the boiling tube should have reached a steady temperature, which will be that of the hot solid. Record this temperature, and that of the water in the calorimeter. The temperature of the calorimeter should be taken with a separate thermometer which should remain in the calorimeter throughout the experiment. Now pour the heated fragments into the calorimeter, and when the temperature has become steady at its highest value, record it. The calorimeter and its contents are then again weighed to find the mass of solid added. The result may be calculated as in the following example :

Mass of calorimeter 88.5 gm.
 Mass of calorimeter and water 148.8 „

∴ Mass of cold water = 60.3 gm.

Initial temperature of water 15°·2 C.
 Initial temperature of hot tacks 98°·4 C.
 Temperature after mixing 25°·3 C.
 Mass of calorimeter and contents after mixing 231.2 gm.

∴ Mass of tacks used = 82.4 gm.

Water equivalent of calorimeter = $88.5 \times 0.1 = 8.8$ gm.

$$(60.3 + 8.8) (25.3 - 15.2) = 82.4s(98.4 - 25.3) \\ s = 0.116$$

The specific heat of other metallic substances, e.g. lead-shot, copper tacks, or granulated zinc can be determined in the same way.

EXPERIMENT 68.—To determine the specific heat of a liquid which does not evolve heat when mixed with water.

If the liquid whose specific heat is required is one which is known not to produce a change in temperature when mixed with water, the specific heat can be determined by adding a known weight of the hot liquid to a known weight of water in a calorimeter. As the specific heat of these liquids is generally considerably greater than that of metals, the mass required for the experiment will not be so large. A volume of liquid equal to about half the volume of the water taken will generally give a suitable rise in temperature. It is not advisable to have too great a rise in temperature in the calorimeter (not more than 20°), as the loss of heat from the calorimeter is much increased if the temperature is high. As a liquid can be stirred and its temperature after stirring taken with a thermometer, it will not be necessary to heat the liquid in a water bath.

Weigh the calorimeter empty, and then when about one-third full of water as in the previous experiment. Heat the liquid in a beaker to about 70° C. (unless this temperature is too near its boiling-point), and remove the burner. Take the temperature of the water in the calorimeter, stir the hot liquid and take its temperature with a separate thermometer. Then without delay pour the hot liquid into the calorimeter, which should then be about half full, stir the mixture well, and read its temperature. The mass of liquid added is then ascertained

by weighing the calorimeter and its contents as before. The result may be calculated exactly as in the previous experiment.

The specific heat of turpentine, paraffin oil, olive oil, and similar liquids can be determined in this way.

Many liquids, however, such as methylated spirit, salt solutions, and so on, evolve or absorb heat when mixed with water. The specific heat of these liquids can be determined by using a solid of known specific heat, and proceeding exactly as in Experiment 67.

EXPERIMENT 69.—To determine the specific heat of a liquid which evolves heat on mixing with water, e.g. methylated spirit.

Weigh a clean calorimeter, pour into it sufficient methylated spirits to fill about one-third of the calorimeter and weigh again. Place a quantity of copper nails in a boiling tube with a thermometer, and heat in a bath of boiling water, exactly as in Experiment 67, for about 10 minutes until the thermometer reading is steady. Then take the temperature of the methylated spirit, add the copper fragments, stir the mixture, and take the steady temperature attained. The weight of copper added can be obtained by re-weighing the calorimeter and its contents. *The specific heat of copper being known* (sp. ht. of copper = 0.094) that of the liquid can be calculated as in the following record :

Mass of calorimeter 88.5 gm.

Mass of calorimeter and spirit 132.7 „

Mass of methylated spirit = 44.2 gm.

Initial temperature of spirit 14°·8 C.

Initial temperature of copper nails 97°·8 C.

Temperature of mixture 27°·4 C.

Mass of calorimeter and contents after mixing = 195.0 gm.

Mass of copper added = 62.3 gm.

Water equivalent of calorimeter = $88.5 \times 0.1 = 8.8$ gm.

Heat absorbed by spirit + heat absorbed by calorimeter.
= heat given out by copper nails.

$$44.2s(27^{\circ}\cdot 4 - 14^{\circ}\cdot 8) + 8.8(27^{\circ}\cdot 4 - 14^{\circ}\cdot 8) \\ = 0.094 \times 62.3(97^{\circ}\cdot 8 - 27^{\circ}\cdot 4)$$

s = specific heat of methylated spirit = 0.62

It must be noted that in making the calculation the heat absorbed by the spirit and that absorbed by the calorimeter must be taken into account separately. The water-equivalent of the calorimeter must on no account be added to the mass of the spirit, as it is when the liquid in the calorimeter is water. The specific heat of other liquids such as a saturated solution of common salt in water can be determined in the same way.

DETERMINATION OF LATENT HEATS

When a solid melts or a liquid vaporises heat is always absorbed in the process, the heat being emitted again when the reverse process takes place. The heat which is absorbed or emitted in the process is called latent heat. The latent heat of fusion of a solid is the quantity of heat required to change unit mass of the substance from the solid to the liquid state without rise in temperature. Similarly, the latent heat of evaporation of a liquid is the heat required to convert unit mass of the liquid from the liquid to the gaseous state without change in temperature. The latent heats of water and steam can be determined as follows :

EXPERIMENT 70.—To determine the latent heat of fusion of ice.

A calorimeter is weighed empty, and again when about one-third full of water. The calorimeter and water are then gently warmed until their temperature is about 5° C. above that of the room. Some ice is then broken into small pieces each about the size of a nut and two or three of these pieces are dried by means of blotting-paper, and then transferred (without touching the ice with the hand) to the water in the calorimeter, the temperature of which has been carefully taken immediately before. The calorimeter is stirred and the lowest temperature reached (*i.e.* when the ice has all melted) is taken. The calorimeter is again weighed, and the increase in weight gives the mass of ice added.

As the temperature of the room is generally above the melting-point of ice, the latter is always melting on the surface and, therefore, rapidly becomes covered with a film of moisture. If the pieces of ice taken for the experiment are too small, it is difficult to dry them properly, and errors will arise from this

source, as, if the ice is wet, the amount of ice added to the calorimeter will be less than the increase in weight by the weight of the water on the surface. On the other hand, if the pieces are too large they will take a long time to melt, and the calorimeter will absorb an appreciable amount of heat from its surroundings during the process.

It is in order to lessen this error that the water in the calorimeter is warmed slightly above the room temperature. During the first part of the melting the calorimeter is losing heat; during the second part, when the ice has cooled the calorimeter below room temperature, it will be gaining heat. The initial temperature of the water and the weight of ice added should be adjusted so that the final temperature of the calorimeter is approximately as far below room temperature as the initial temperature is above it.

The final temperature must not be so low that dew is deposited on the calorimeter, or very serious error will be caused.

The quantity of heat absorbed from the warm calorimeter and water is used up in melting the ice, and in raising the temperature of the melted ice to the final temperature of the calorimeter. Thus, if L is the latent heat of fusion, M_1 the mass of warm water, and t_1 its initial temperature, while M_2 is the mass of ice added,

$$(M_1 + W)(t_1 - t_2) = M_2 L + M_2(t_2 - 0)$$

where W is the water equivalent of the calorimeter and t_2 the final temperature. The following example illustrates the method of calculation :

Mass of calorimeter 56.2 gm.

Mass of calorimeter and warm water . . 168.4 ,,

\therefore Mass of water = 112.2 gm.

Water equivalent of
calorimeter = 5.6 gm.

Initial temperature of calorimeter . . . 20°·4 C.

Final temperature of calorimeter . . . 10°·0 C.

Mass of calorimeter, etc., after experiment 181.8 gm.

\therefore Mass of ice used = 181.8 - 168.4 = 13.4 gm.

$(112.2 + 5.6)(20^\circ\cdot4 - 10^\circ) = 13.4 \cdot L + 13.4(10^\circ - 0)$

$L = 79.7$ calories per gm.

EXPERIMENT 71.—To determine the latent heat of evaporation of water. (Latent heat of steam.)

The accurate determination of the latent heat of evaporation is a very difficult process, owing to the number of errors which have to be avoided. In the first place, owing to the great difference in temperature between the steam and the laboratory, partial condensation of the steam is very apt to occur in the tube leading from the boiler to the calorimeter, if the tube is at all long, while if the boiler is placed too near the calorimeter

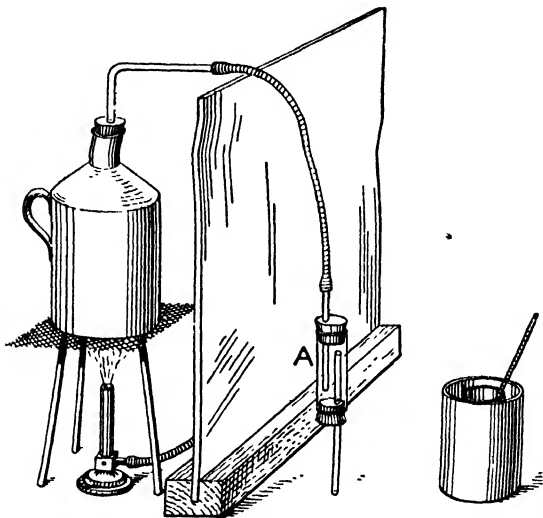


FIG. 46.—Determination of the Latent Heat of Steam.

the latter may be appreciably heated by radiation from the hot boiler. To reduce these errors a steam-trap, A (Fig. 46), may be constructed and attached to the steam delivery tube, just before it dips into the calorimeter. The efficiency of this trap is increased if it is wrapped in flannel or cotton-wool (not shown in the figure) to prevent condensation in the trap itself. The trap may easily be made from a piece of glass tubing, about 6 cm. long and about 2 cm. diameter, and a couple of well-fitting corks.

Fit together the apparatus as shown in the figure, and commence heating the water in the boiler. Weigh the calorimeter

meter empty, and about one-half full of water. When the steam is issuing freely from the exit tube, take the temperature of the water in the calorimeter, and insert the nozzle of the steam tube well under the surface of the water in the calorimeter. Allow the steam to pass into the calorimeter until the temperature has risen to about 45°C . Withdraw the nozzle quickly, stir the calorimeter well, and record the highest temperature reached. The calorimeter is then weighed again to determine the mass of steam condensed. If M_1 and W are the mass of water taken and the water equivalent of the calorimeter, t_1 the initial temperature, and t_2 the final temperature of the calorimeter, and M_2 the mass of steam condensed, then the heat given out by the steam in condensing to water at 100°C ., plus the heat given out by this water in cooling from 100°C . to t_2 , is equal to the heat taken in by the calorimeter and cold water. Thus

$$M_2 L + M_2(100 - t_2) = (M_1 + W)(t_2 - t_1)$$

The following is a record of an actual experiment:

Mass of calorimeter 56.2 gm.

Mass of calorimeter and water 192.6 ,,

\therefore Mass of water = 136.4 gm.

Water equivalent of calorimeter = 5.6 gm.

Initial temperature $14^{\circ}.4\text{ C.}$

Final temperature $44^{\circ}.6\text{ ,,}$

Final mass of calorimeter and contents 199.8 gm.

\therefore Mass of condensed steam = $199.8 - 192.6 = 7.2\text{ ,,}$

$$7.2 \cdot L + 7.2(100 - 44^{\circ}.6) = (136.4 + 5.6)(44^{\circ}.6 - 14^{\circ}.4)$$

$$L = 541\text{ calories per gm.}$$

The correct value is 536 calories per gm. A source of error in this form of the experiment is that water is apt to be withdrawn on the nozzle of the steam pipe when the latter is removed from the calorimeter. This leads us to under-estimate the amount of steam condensed. As the mass of steam condensed is generally small, the loss of a single drop in this way may cause an appreciable error in the result. This may be avoided by passing the steam not directly into the calorimeter, but into a small copper condenser immersed in the calorimeter. The water equivalent of the condenser must, of

course, be added to that of the calorimeter itself in making the calculations. Using a condenser an accuracy of within 3 per cent. can be obtained. When the steam is passed directly into the water the error may amount to as much as 6 per cent. even with a careful experimenter. With possible errors of this magnitude, it is obviously unnecessary to make any allowance for change in boiling-point of water with change in barometric pressure.

§ 22. EXPERIMENTS OF COOLING

DETERMINATION OF SPECIFIC HEAT BY THE METHOD OF COOLING

THE rate at which a given vessel, such as a calorimeter, loses its heat depends on the nature and the area of the surface of the vessel, and on the difference in temperature between the vessel and its surroundings. The same vessel will, however, always lose heat at the same rate if its surface and the difference in temperature between it and its surroundings remains the same. If a calorimeter of water equivalent w , contains a mass m_1 of water, and takes x seconds to cool from a temperature t_1 to a lower temperature t_2 , the heat given out is $(m_1 + w)(t_1 - t_2)$ calories, and its average rate of cooling over this interval of temperature is $\frac{(m_1 + w)(t_1 - t_2)}{x}$ calories per sec.

If when the water is replaced by a mass m_2 of liquid of specific heat s , the calorimeter is found to cool from the same temperature t_1 to the temperature t_2 in y seconds, the loss of heat is now $(m_2s + w)(t_1 - t_2)$, and the rate of loss of heat is $\frac{(m_2s + w)(t_1 - t_2)}{y}$. But these rates must be the same since the vessel and the difference in temperature between the vessel and its surroundings is the same in both cases. Hence

$$\begin{aligned} \left(\frac{m_1 + w}{x}\right)(t_1 - t_2) &= \left(\frac{m_2s + w}{y}\right)(t_1 - t_2) \\ s &= \left(\frac{m_1 + w}{m_2}\right)\frac{y}{x} - \frac{w}{m_2} \end{aligned}$$

EXPERIMENT 72.—To determine the specific heat of a liquid by the method of cooling.

The calorimeter used for cooling experiments should, if possible, be fitted with a lid of the same metal as the rest of the calorimeter, and should have three small hooks soldered

to it near the top, so that it can be suspended by strings inside a larger metal vessel. This outer vessel should be placed in a large pan of water at the temperature of the room, as shown in Fig. 47. This will, by its large thermal capacity, ensure that the temperature of the outer vessel remains reasonably constant during the course of the experiment.

To read the temperature of the inner calorimeter a small hole is made in the lid, through which a thermometer can be passed. A second small hole can be made to allow of the use of a small stirrer. A more convenient method, however, is to make a little brass vane, such as shown in Fig. 47, A, attached to a split tube, which clips on to the lower end of the thermometer. With this attachment the thermometer can also be

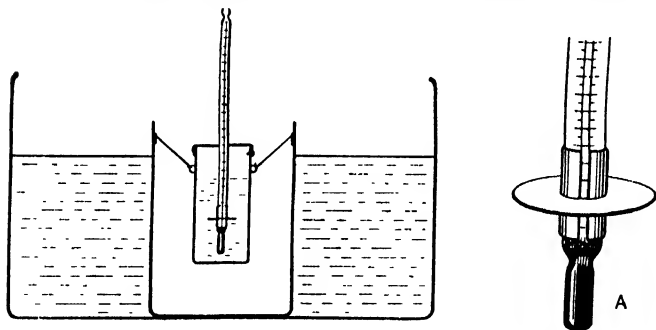


FIG. 47.—Calorimeter for Experiments on Cooling.

used as a stirrer, care being taken not to stir so vigorously as to break the thermometer bulb.

To carry out the experiment the calorimeter is weighed, and its water equivalent calculated. It is then suspended inside the outer vessel by hooking it to the strings, and a quantity of water at a temperature of about 70°C . is poured into the calorimeter, sufficient to fill it about three-quarters full. The thermometer and the lid are then fitted on, and the water is gently stirred. By means of a clock or watch with a seconds hand, note the exact time when the temperature reaches 60°C . and continue stirring until the thermometer falls to 55°C ., when the time is again taken. Continue the experiment until the temperature falls to 50°C ., when the time is again recorded. Remove the calorimeter from the outer vessel, and weigh again to determine the mass of water present.

Pour the water away, dry the calorimeter, and again suspend it in the outer vessel, and pour into it a volume of the liquid whose specific heat is to be determined, and which has been heated to about 70°C. or 75°C. The volume of the liquid used should be approximately the same as that of the water, that is to say, it should about three parts fill the calorimeter. Proceed exactly as in the case of the water, taking the time of cooling from 60°C. to 55°C. , and from 55°C. to 50°C. These times will probably be considerably shorter than in the case of the water. The mass of liquid used is obtained by weighing the calorimeter and its contents at the close of the experiment. The specific heat of the liquid can be calculated as in the following example of a determination of the specific heat of glycerine :

Mass of calorimeter (copper) = 52.2 gm. \therefore Water equivalent of calorimeter = 5.2 ,,					
COOLING EXPERIMENTS. •					
With Water.			With Glycerine.		
Temp.	Time.	Time to Cool 5°.	Temp.	Time.	Time to Cool 5°.
60° C.	11h. 5m. 30s.	5m. 15s.	60° C.	11h. 25m. 20s.	3m. 45s.
55° C.	11h. 10m. 45s.		55° C.	11h. 29m. 5s.	
50° C.	11h. 16m. 35s.		50° C.	11h. 33m. 20s.	
Mass of calorimeter and water } = 104.6 gm.			Mass of calorimeter and glycerine } = 114.5 gm.		
\therefore Mass of water = 52.4 ,,			\therefore Mass of glycerine = 62.3 ,,		
For the temperature interval 60° to 55°					
$\frac{(52.4 + 5.2)5}{315} = \frac{(62.3 + 5.2)5}{225}$					
$s = 0.578.$					
For the temperature interval 55° to 50°					
$\frac{(52.4 + 5.2)5}{350} = \frac{(62.3 + 5.2)5}{255}$					
$s = 0.592.$					
\therefore Mean specific heat of glycerine = 0.585.					

EXPERIMENT 73.—To investigate the law of cooling.

For this experiment the calorimeter used in the previous experiment may be employed. It is weighed and suspended, as before, inside an outer metal vessel surrounded by a pan of water at the temperature of the laboratory. Sufficient hot water at a temperature of about 75°C. to 80°C. is poured into the calorimeter to about three parts fill it, and the lid is replaced. The water is now gently stirred, the stirring being continued throughout the experiment. Wait for a couple of minutes after

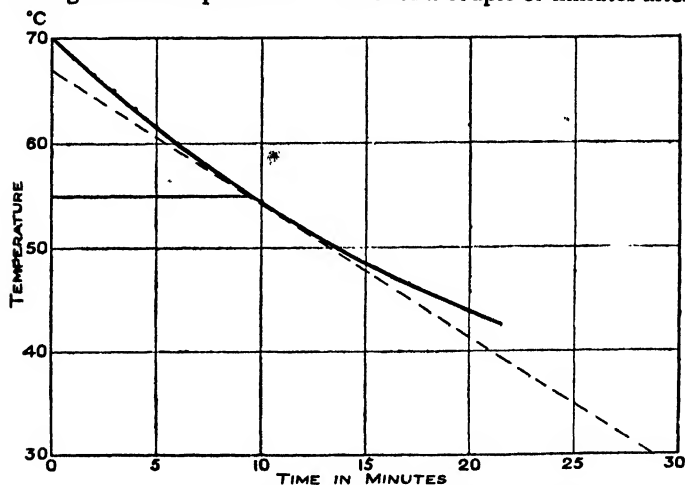


FIG. 48.—Cooling Curve for Water. The dotted line is the Tangent to the Curve at 55°C. , and indicates the Method of deducing the Rate of Cooling.

pouring the water into the calorimeter in order to allow the calorimeter to heat up to the temperature of the water, and then take the temperature of the water every minute, recording the time and the temperature in two parallel columns. Continue the observations until the cooling becomes too slow to be measured conveniently, say, down to a temperature of 40°C. By weighing the calorimeter at the end of the experiment the mass of water used may be ascertained.

A cooling curve may now be plotted on squared paper, the temperatures on the vertical axis, and the corresponding times along the horizontal axis. Owing to difficulties of reading the thermometer accurately the experimental points may not lie exactly on a smooth curve. These slight irregularities may be

neglected and a smooth curve drawn so as to lie as closely as possible to the experimental points. This will be the cooling curve for the calorimeter.

The rate of fall in temperature at any given point on the curve is equal to the slope of the tangent to the curve at that point, and may be deduced from the curve (Fig. 48) by the method described in Experiment 30. The rate of loss of heat from the calorimeter is calculated by multiplying this quantity by the thermal capacity of the calorimeter and its contents, that is, by the mass of water plus the water equivalent of the calorimeter. Calculate in this way the rate of cooling of the calorimeter at temperatures of, say, 65°C. , 60°C. , 55°C. , and so on, and tabulate the results.

According to Newton's Law of Cooling, the rate of loss of heat is proportional to the difference in temperature between the hot body and its surroundings, in this case the water-bath. Plot a second graph between the rate of cooling of the calorimeter and its excess of temperature over that of the bath. This curve should be approximately a straight line, showing that, for the comparatively small intervals of temperature used in our experiment, Newton's law is at any rate approximately true. The method of calculating may be seen from the following record of an experiment :

Mass of calorimeter and stirrer . . . 52.6 gm. Water equivalent 5.3 „ Temperature of water-bath 13°·3 C.			
Time in Minutes.	Temp. of Calorimeter in ° C.	Time in Minutes.	Temp. of Calorimeter in ° C.
0	70.0	11	53.0
1	68.0	12	51.9
2	66.5	13	50.6
3	65.0	14	49.5
4	63.4	15	48.4
5	61.8	16	47.5
6	60.0	17	46.7
7	58.4	18	45.7
8	57.0	19	44.8
9	55.5	20	43.9
10	54.2		
Mass of calorimeter and water = 108.6 gm. ∴ Mass of water = 56.0 „ Thermal capacity of calorimeter and contents = 56.0 + 5.3 = 61.3 gm.			

TANGENTS DRAWN TO COOLING CURVE AT 65°, 60°, 55°, 50°, AND 45°.						
Tangent at	Intercept made by Tangents on		Slope of Tangent = Temp. Change per min.	Rate of Loss of Heat Cal. per min.	Difference between Cal. and Bath in ° C.	Rate of Cooling ÷ Temp Diff.
	Temp. Axis.	Time Axis.				
65°	69°·5 - 30°	24·6 min.	$\frac{39\cdot5}{24\cdot6} = 1\cdot61$	98·6	51°·7	1·91
60°	69°·0 - 30°	24·9 „	$\frac{39\cdot0}{24\cdot9} = 1\cdot56$	95·6	46°·7	2·04
55°	67°·2 - 30°	28·5 „	$\frac{37\cdot2}{28\cdot5} = 1\cdot31$	80·3	41°·7	1·92
50°	64°·6 - 30°	32·1 „	$\frac{34\cdot6}{32\cdot1} = 1\cdot07$	65·6	36°·7	1·79
45°	62°·1 - 30°	35·0 „	$\frac{32\cdot1}{35\cdot0} = 0\cdot92$	56·4	31°·7	1·78

The rate of loss of heat from a surface depends on the nature of the surface. Thus a lamp-black surface radiates heat much more rapidly than a brightly polished one. We can compare the radiating powers of two surfaces in the following way :

EXPERIMENT 74.—To compare the radiating powers of two surfaces.

Polish the calorimeter used in the previous experiment. Pour into it a measured volume of hot water, and obtain the cooling curve as already described. Then empty out the water, and blacken the outer surface of the calorimeter by holding it in a smoky flame. Suspend it as before, and pour in the same measured volume of hot water, and obtain a second cooling curve. This will be found to be steeper than before. The ratio of the slopes of the tangents to the two curves at the same temperature will be proportional to the radiating powers of the surfaces.

Instead of blackening the calorimeter, a second calorimeter of the same dimensions as the first, but having its surfaces coated with a dead-black varnish, can be used.

ADDITIONAL EXERCISES AND EXAMINATION QUESTIONS.—III

1. Determine the change in temperature which takes place when 10 gms. of powdered sodium hyposulphite are added to 50 gms. of water at a temperature of 20°C .

2. Determine the boiling-point of the given liquid (*e.g.* a solution of equal parts by weight of calcium chloride in water) by a calorimetric method.

(Heat a mass of metal of known specific heat in the boiling liquid. Transfer to a calorimeter, and from the heat given out by the metal deduce its initial temperature.)

3. Determine the temperature of the laboratory by means of the given constant volume air thermometer. No other thermometer is supplied.

4. *Determine the temperature of the given brass cylinder* after it has been heated to the temperature of boiling water, and then allowed to cool in the air for three minutes.

5. Using a copper calorimeter and copper nails, compare the specific heats of the given liquid and copper.

6. Heat a calorimeter containing water in a steady bunsen flame. Plot a curve showing how the temperature varies with the time since the heating began.

7. Assuming that the specific heat of the given wax is 0.6 in both solid and liquid forms, find the latent heat of fusion of the wax.

(The wax is heated to a measured temperature above its melting-point and poured into a weighed amount of water in a calorimeter. The heat given out by the wax in cooling and solidifying is $Ws(t_1 - t_2) + WL$ where W = the weight of wax, and t_1 and t_2 are the initial and final temperatures of the wax.)

8. Compare the rate of loss of heat from a blackened and a polished calorimeter of the same dimensions at temperatures of 70° and 50° .

BOOK IV

LIGHT AND SOUND

§ 23. PHOTOMETRY

COMPARISON OF ILLUMINATING POWERS

THE standard of illuminating power is that of a candle of sperm wax, of which six go to the pound, and which burns at the rate of 120 grains per hour. The number of such candles which would be required to produce the same intensity of illumination as that produced by a given source of light at the same distance from the source is called the candle power of the source. The eye is not able to estimate with any accuracy how many times one source is brighter than another. It can, however, distinguish with an accuracy which amounts in favourable cases to within 1 or 2 per cent. whether two adjacent parts of the same screen are of equal brightness. Since the brightness or intensity of illumination of a screen is inversely proportional to the square of the distance from the source of light, this fact can be employed for comparing the illuminating powers, or candle powers, of two different sources. Suppose P_1 and P_2 are the illuminating powers of two sources of light, and d_1 and d_2 are the distances at which they produce equal illumination on a given screen, the intensity of illumination of the screen due to the first source is $\frac{P_1}{d_1^2}$ and that due to

the second is $\frac{P_2}{d_2^2}$. If these are equal we have

$$\frac{P_1}{d_1^2} = \frac{P_2}{d_2^2} \quad \text{or} \quad \frac{P_1}{P_2} = \frac{d_1^2}{d_2^2}$$

The illuminating powers of two sources of light are directly proportional to the squares of the distances at which they produce equal intensities of illumination.

This result is applied in various ways. One of the simplest is the Rumford photometer.

EXPERIMENT 75.—To compare the illuminating powers of a lamp and candle by Rumford's photometer.

All photometric experiments should be performed in a darkened room, as any light falling upon the screen, either directly from the window, or even reflected from the walls of the room, may interfere seriously with the experiment. If a dark room is not available, the darkest part of the laboratory should be selected for the experiment, and the screen itself protected as far as possible from direct light by a large drawing-board.

A white screen of stout cardboard B (Fig. 49) is fastened to

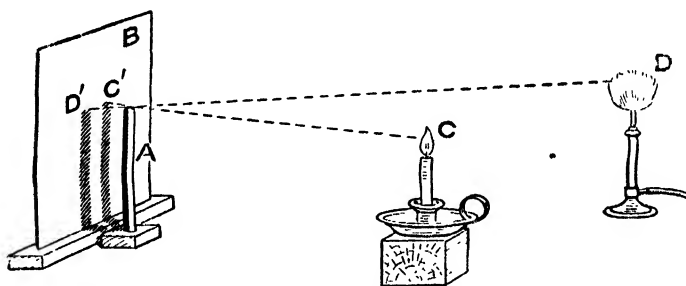


FIG. 49.—Rumford's Photometer.

a wood block so that it stands vertically on the bench, and a vertical cylindrical rod, such as a small retort stand, is placed in front of the screen. A lighted candle is placed about 30 cm. from the screen, and the light whose illuminating power is to be compared with that of the candle is also placed in front of the screen at a somewhat greater distance. A very convenient source of light for optical experiments may be obtained by screwing an ordinary fish-tail burner into the top of a bunsen burner, as shown in the figure. If necessary, the candle may be supported on a wood block so that the flames are at the same height above the bench. Two shadows of the rod will be seen on the screen, the one, D', thrown by the gas flame, the other, C', by the candle. The shadow cast by the candle is illuminated by the gas flame, while that cast by the flame is illuminated by the candle. Alter the distance of the gas

burner from the screen until the two shadows are of the same depth. This adjustment will be facilitated if the flames are moved sideways until the two shadows just touch each other without overlapping. When the two shadows are of the same depth the intensities of illumination produced on the screen by the two sources is the same, and $\frac{P_1}{P_2} = \frac{d_1^2}{d_2^2}$ where d_1 and d_2 are the distances of the two sources from the screen. These distances are measured with a metre scale. The setting should be repeated several times, and the mean value of $\frac{P_1}{P_2}$ taken.

If the two sources are of approximately the same colour, as in the case of a candle and an oil lamp, for example, the observed values of the ratio should agree fairly closely. If, however, there is a considerable difference in the colour of the light given out by the two sources, as is the case when we attempt to compare the illuminating power of a metallic filament electric lamp with that of a candle, the two shadows will be of a noticeably different tint, that illuminated by the candle being distinctly yellowish, and comparison becomes more difficult. As far as possible this difference in colour must be ignored, and the shadows judged entirely by their depth, and not by their colour. This difference in colour of the shadows is the most serious source of error in photometric experiments.

EXPERIMENT 76.—To verify that the intensity of illumination is inversely proportional to the square of the distance by means of a Bunsen photometer.

A Bunsen, or grease-spot, photometer (Fig. 50) can easily be made as follows: In the centre of a square of cardboard (about 8 cm. square) cut a large circular hole, and gum over it a piece of unglazed writing-paper. On the centre of this drop a single spot of melted grease from an ordinary candle, and when it has set hard remove all the superfluous grease with a knife. If the paper is placed between your eye and the window the spot will appear brighter than the rest of the paper; if you stand with your back to the window and hold the paper against a dark background, the spot appears dark. If the paper is equally illuminated from both sides, the spot is almost indistinguishable from the rest of the paper.

Lay a metre scale on the table and clamp the photometer in a vertical position at the centre of the scale. On one side

of the photometer place a single lighted candle at a distance of about 20 cm. from the screen, and on the other side four similar candles placed side by side and as close together as possible. Move these backward or forward until the grease spot disappears as completely as possible when viewed from either side of the screen. Measure the distances of the single candle and of the four candles from the photometer. If the adjustment is correct, the distance between the cluster of candles and the screen will be found to be twice that between the screen and the single candle. Repeat the experiment several times for different distances of the single candle from the photometer.

The adjustment of the photometer is made easier if a pair of mirrors are placed on one side of the photometer so that

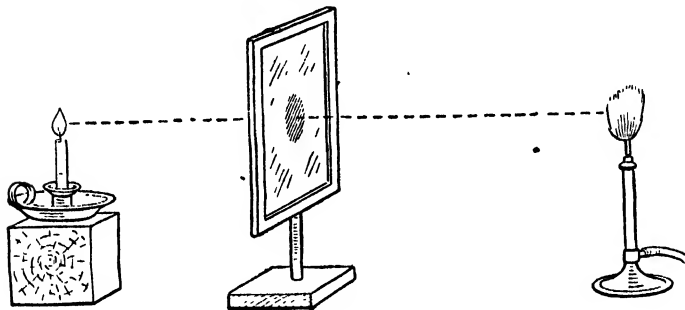


FIG. 50.—The Bunsen Photometer.

both sides of the photometer can be viewed simultaneously. The photometer is then adjusted until the contrast between the grease spot and the cardboard is the same on both sides.

By replacing the four candles by a gas burner, the illuminating power of the burner and the candle can be compared. If P_1 and P_2 are the burner and the candle, and d_1 and d_2 their respective distances from the screen when the grease spot becomes invisible,

$$\frac{P_1}{P_2} = \frac{d_1^2}{d_2^2}$$

It should be noted that the presence of stray light in the room has a much more disturbing effect on the Bunsen than on the Rumford photometer. In the latter, the two shadows are cast side by side and will be equally illuminated by stray light in the room. Thus, so long as the latter does not interfere

with the visibility of the shadows, no error will arise. In the case of the grease-spot photometer, the intensity of the light in the room is almost certain to be different on the two sides, and may cause a large error. The grease-spot photometer will only give accurate results in a dark room.

§ 24. REFLEXION AND REFRACTION

LAWS OF REFLEXION OF LIGHT

THE incident ray, the reflected ray, and the normal to the reflecting surface at the point of incidence all lie on one plane.

The incident ray and the reflected ray make equal angles with the normal at the point of incidence.

EXPERIMENT 77.—To verify the laws of reflexion of light for a plane surface.

For this experiment a plane mirror will be required. If procurable, a mirror silvered on the front surface is the most

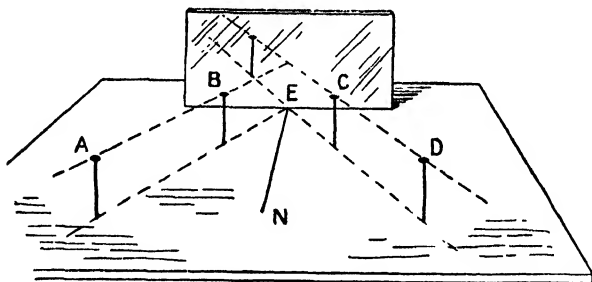


FIG. 51.—Verification of the Laws of Reflexion, using a Plane Mirror.

suitable, but if a mirror silvered in the usual way on the back surface is used the glass should be as thin as possible. If a thick plate glass mirror is used, the refraction which takes place in the glass will appreciably affect the results of the experiment. The mirror is placed vertically in the centre of a horizontal drawing-board covered with white paper, and the position of the reflecting surface is marked on the paper by drawing a line along it with a sharp pencil.

Stick a pin A (Fig. 51), upright in front of the mirror, and a second pin, B, some 10 or 12 cm. away from it, so that the straight line joining the two pins meets the mirror obliquely.

On looking into the mirror, images of the two pins will be seen. Close one eye and move your head until the image of A and of B appear in the same straight line. Keeping your head in the same position stick in two more pins, C and D, so that these pins and the images of A and B appear to be exactly in line. Remove the mirror and the pins, and join the pin-pricks left by A and B and by C and D by straight lines. These should meet on the line marking the position of the reflecting surface, at some point E. With a set square draw the normal EN at E, and measure the angles AEN and DEN. These are the angles of incidence and reflexion.

Since light travels in straight lines, the line AB represents the path of a ray of light which proceeds from A and passes through B. Since when looking in the direction DC we see the four pins, ABCD, apparently in the same straight line, C and D are also points on the same ray after reflexion at the mirror. Hence BAECD is the path of the ray of light, and the angles BEN and DEN are the angles of incidence and reflexion.

Replace the mirror on the same line as before, and repeat the experiment, moving B sideways, so that the line BA makes a different angle of incidence with the mirror. Make four or five such observations and record the corresponding angles of incidence and reflexion in two parallel columns. The angle of incidence should be equal to the angle of reflexion in each case.

If, when the pins are in position, they are pressed into the drawing-board so as to be all exactly at the same height above the board, it will be found that, on looking along the line CD, all the pin-heads appear exactly in line. But the pin-heads are all in a plane which is at right angles to the plane of the mirror (since the mirror is at right angles to the drawing-board). Thus the first law of reflexion is verified.

OPTICAL IMAGES

If a pencil of rays starting from a point on an object are after any number of reflexions and refractions made to pass through some other point, or to appear to pass through some other point, the second point is called the optical image of the first point.

If the rays actually pass through the image, the latter is said to be real, and can be caught on a screen placed at that point.

If the rays only appear to pass through the point without actually doing so, the image is said to be virtual.

The position of a virtual image can be determined in various ways which we shall have to consider.

On looking into a plane mirror (looking-glass) we see an image of any object placed in front of it. This image appears to be behind the mirror. As the rays of light from the object do not pass through the mirror but are reflected from it according to the laws of reflexion (Experiment 77), it is evident that they do not actually pass through any point in the image, but only appear to do so to the eye. The image is, therefore, a virtual image.

It can be easily deduced from the laws of reflexion that the image is as far behind the surface of the mirror as the object is in front of it. We can also demonstrate this experimentally as follows :

EXPERIMENT 78.—To prove that the rays of light proceeding from an object form an image as far behind the mirror as the object is in front of it.

Proceed exactly as in Experiment 77 to trace the path of a ray

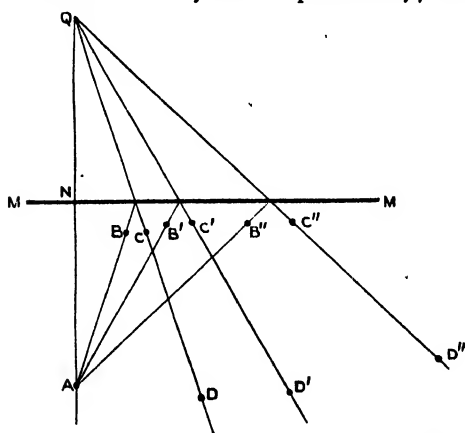


FIG. 52.—Determining the Position of the Image of an Object in a Plane Mirror, by Tracing Rays.

of light, ABCD, reflected in the mirror. Keeping the pin A fixed to serve as the object, move the pin B a little to one side to some position such as B' and trace the path of the ray AB'C'D'. Move the pin B a little further still and trace the new ray, AB''C''D''. Do this for three or four positions of

the pin B, so as to obtain the paths of several rays, all starting from the object A, and reflected from the mirror surface.

Now produce the reflected rays, CD, C'D', etc., backwards

behind the mirror. If the experiment has been carefully performed, all these rays will be found to pass through a single point, Q, behind the mirror. Thus, since all the rays starting from A after reflexion in the mirror appear to come from a point Q, Q is the image of A, and since the rays do not actually pass through Q, the image is virtual.

Join A to its image Q by a straight line, cutting the reflecting back surface of the mirror in N. The line AQ will be found to be at right angles to the mirror surface MM. Measure the distances AN and QN with a steel scale. They will be found to be equal.

The image of an object in a plane mirror is as far behind as the object is in front of the mirror.

PARALLAX

A most important method of finding the position of an image, whether real or virtual, is that known as the method of parallax. Parallax may be illustrated by the following experiment:

EXPERIMENT 79.—To illustrate the method of parallax.

Take two pins and clamp them in suitable clamps so that one is pointing vertically upwards, and the other vertically downwards. Their points should be at the same height above the bench so that they just touch when the pins are placed together. Place one pin about 10 cm. in front of the other. By closing one eye and suitably placing your head it will be possible to see the two pins in exactly the same line. If, however, the head is moved to the left of this position the nearer pin will appear to be to the right of the other; if the head is moved to the right it will appear to move to the left. This relative motion is known as parallax. If the distance between the pins is reduced, the parallax will become less, until when the pins are exactly in line, point to point, there will be no parallax, and the two pins will be seen in one straight line no matter where the head is placed. Conversely if the two pins always keep in the same line no matter in what direction they are viewed then they must be in the same position.

This is equally true if A is the image of a pin, instead of a real pin. To find the position of an image, therefore, we

can take a search-pin and move it about until it appears to be in line with the image, from no matter where we view it. When this adjustment has been made the search-pin is in the position of the image we are seeking. To assist in making the adjustment we may remember that the nearer of the two will appear to move in the opposite direction to that in which the head is moved.

EXPERIMENT 80.—To find the position of the image formed in a plane mirror by the method of parallax.

Place the mirror in an upright position. At a distance of about 10 cm. in front of the mirror place one of the pins, and place the other behind the mirror. The second pin should be sufficiently long to be seen easily over the top of the mirror.

Move the pin B until the part of it which projects above the mirror appears to be in the same straight line as the image of the pin A in the mirror (see Fig. 53). Now move the head a foot or so to one side. The two will no longer appear to be in line.

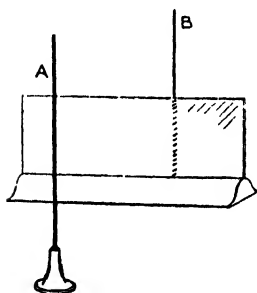


FIG. 53.—To find the Image of an Object in a Plane Mirror by the Parallax Method.

Remember that whichever of the two appears to move relatively to the other in the same direction as that in which you have moved your head is the further away from you.

If B appears to move relatively to the image of A in the mirror in the direction in which you have moved your head, it is too far behind the mirror and should be moved a little closer.

Adjust the position of B until on moving your head from side to side the image of A and the pin B remain in the same straight line. B is then in the position of the image of A in the mirror. Measure the distances of A and B from the mirror surface. This can be done roughly by means of a millimetre scale, placed so as to touch both A and B and resting lightly against the top edge of the mirror. If this method is used, great care must be taken that neither the pins nor the mirror are displaced by the scale.

A more accurate way is to use a large pair of dividers (Fig. 1).

Open the dividers until the two points will just reach from A to the mirror, or from B to the mirror respectively. Then place the dividers carefully on a millimetre scale and read off the distance between the points. If the points are sharp it will be possible to read off this distance very accurately, since fractions of a millimetre can be estimated by the eye.

Measure the distances of A and B from the mirror by both these methods and record the results in your note-book. The experiment should be repeated several times, with A at different distances from the mirror.

At first some difficulty may be found in fixing the true position of the image of A by this method. The experiment should be repeated until it can be performed with reasonable certainty. The following hints may be useful:

Use fine pins. It is almost impossible to get good results if the pins are thick and clumsy.

Always arrange the experiment so that the image of A and the finding-pin B are exactly in the same straight line, as shown in the picture. A small change in the distance between two lines is not easily estimated, but a very slight shift in one of the two parts of a straight line is easily detected.

If the mirror used is not made of good glass it is very possible that the different parts of it will not be accurately plane. In this case the image will be seen to wobble from side to side as the head is moved along. Accurate readings will then be impossible and a better piece of mirror must be sought for.

The results should be recorded as follows :

Distance of Object Pin from Back of Mirror.	Distance of Image Pin from Back of Mirror.	Difference.
6.20 cm.	6.15 cm.	- 0.05 cm.
12.45 "	12.55 "	+ 0.10 "
18.70 "	18.60 "	- 0.10 "
Mean Difference		- 0.017 cm.

LAWS OF REFRACTION OF LIGHT

The incident ray and the refracted ray lie on opposite sides of the normal at the point of incidence but in the same plane with it.

The sine of the angle of incidence bears to the sine of the angle of refraction a ratio which is constant for the same pair of optical media.

EXPERIMENT 81.—To verify the laws of refraction of light.

The laws of refraction of light can be verified by tracing the paths of rays of light through some suitable transparent substance. The most convenient is a rectangular block of

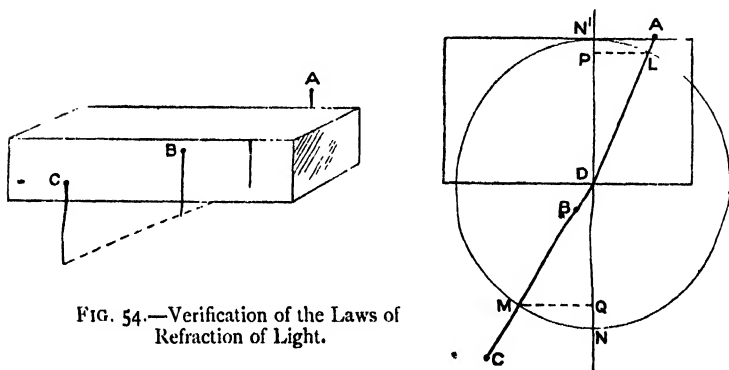


FIG. 54.—Verification of the Laws of Refraction of Light.

glass, such as is sometimes used for a paper weight.* For accurate results to be obtained the block should not be less than 6 cm. thick.

* The block is placed on a sheet of drawing paper on a drawing-board, and lines are drawn round its edges with a sharp pencil so as to mark the positions of the surfaces. A pin is then placed in contact with the back surface of the block and near one edge of it. A second pin is then placed in front of the block. The head is then placed so that the pin B appears in line with the image of the pin A as seen looking through the glass. Keeping the head in this position a third pin C is placed so that the three appear in one and the same line. Now move the pin B a centimetre or so to the left or right of its original position to some point B' and repeat the experiment. Do this for three or four different

positions of the pin B. The block can then be removed and the points CB, C'B', . . . joined by straight lines which are produced to meet the front surface of the block in the points D, D', . . . (Fig. 54).

Now ADBC is obviously the path of a ray of light starting from A and passing through the block. Similarly AD'B'C' represents another ray, and so on. With D as centre describe a circle of about 8 cm. radius cutting AD and DC in points M and L. By means of a set square draw the normal to the surface of the block at D, and drop perpendiculars from L and M on this normal meeting it in P and Q. The law of refraction states that for a given pair of media (glass and air in this case) the ratio of the sine of the angle NDC to the sine of the angle N'DA should be constant. But in our diagram $\frac{\sin \text{NDC}}{\sin \text{N'DA}}$ is equal to $\frac{\text{MQ}/\text{MD}}{\text{PL}/\text{LD}} = \frac{\text{MQ}}{\text{PL}}$, since MD = LD.

Measure the perpendiculars PL and MQ and calculate the ratio $\frac{\text{QM}}{\text{PL}}$. Repeat the construction for the other rays. The ratio should be constant. It measures the refractive index of glass relative to air. The results may be recorded as follows:

QM.	PL.	$\frac{\text{QM}}{\text{PL}} = \mu.$
2.52	1.64	1.54
3.75	2.48	1.51
5.28	3.48	1.52
6.13	4.05	1.51
Mean . .		1.52

§ 25. EXPERIMENTS WITH MIRRORS AND LENSES

REAL AND VIRTUAL IMAGES

THE images formed by mirrors and lenses may be of two kinds. In some cases the rays of light diverging from a point on the object have their paths diverted by the mirror or lens in such a way that they all come to a focus at some second point, which is known as the optical image of the first point. Since the rays actually pass through the second point they can be caught on a screen placed at that point, and an image of the object will be seen on the screen. This is known as a real image. The position of a real image can be found by moving a white screen in the path of the rays until the image appears sharply focused on the screen. It can also be determined by the method of parallax. The former method is perhaps the easier, though it requires a luminous object and a room which is partly darkened so that the image may be plainly visible on the screen. For the parallax method an ordinary pin will serve as the object, and the room should be as light as possible. The parallax method is capable of greater accuracy than the focusing method, as the search-pin can be set by parallax in the position of the image with greater certainty than the image can be focused on a screen. With practice the position of a real image can be determined by the parallax method to within a millimetre.

In some cases the rays leaving the object are not brought to a point again, but are deflected so that they appear to be radiating from some second point. This point is called a virtual image of the object. As the rays do not pass through the virtual image the latter cannot be obtained on a screen. The position of the virtual image can be determined by the method of parallax.

In the experiments which follow, the position of the real images may be determined either by the screen method or by

the parallax method. For the former an ordinary flame does not provide a sufficiently definite object for the experiment, and its distance from the lens or mirror cannot be measured with sufficient accuracy. A suitable object can be made by placing a piece of wire gauze in front of the flame and 1 or 2 cm. away from it. This may conveniently be soldered to a clip which clips on to the burner as shown in Fig. 55, *a*. A piece of tin plate (Fig. 55, *b*), having a circular hole 1 cm. in diameter across which are soldered fine cross wires can also be used. For focusing the image a white cardboard screen, mounted on a suitable stand, or a piece of ground glass, such as is used in focusing cameras, is convenient.

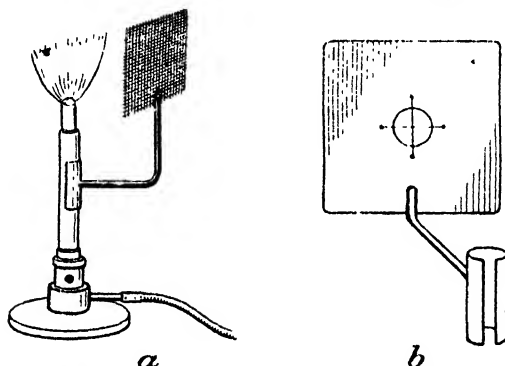


FIG. 55.—Methods of obtaining Suitable Luminous Objects for Optical Experiments.

For the parallax method a couple of needles or fine pins, mounted on stands, are all that will be required. One pin serves as the object, the other as the search-pin to locate the image.

FORMULÆ FOR SPHERICAL MIRRORS AND LENSES

Let u be the distance of the object from the mirror or lens, and v the distance of the corresponding image. These distances must always be measured from the mirror or lens, and distances measured in the opposite direction to that in which the light is travelling must be considered as positive,

while those measured in the same direction as that in which the light is travelling must be regarded as negative. Thus if the object and image are both on the same side of the mirror or lens both will have the same sign (+). If the image is on the opposite side of the mirror or lens to the object, u will be positive (if we are dealing with a real object) and v will be negative. Using this convention, which is quite simple to understand and remember, we have

For all spherical mirrors

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

For lenses

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

On this convention the focal length, f , of a concave mirror or a concave lens will always be positive; that of a convex mirror or a convex lens will be negative.

The elementary formulæ for mirrors and lenses are only true for points very near the axis of the mirror or lens, and care should be taken in making the experiments that this condition is fulfilled—that is to say, that the mirror or lens is at right angles to the line joining the image and the object. This setting is facilitated by the use of some form of optical bench, but it can be done sufficiently accurately by inspection. When the screen method is employed for concave mirror experiments it will be necessary to have both screen and lens a little to one side of the axis of the mirror so that they may not get in each other's way. They should, however, be kept as near the axis as possible.

EXPERIMENT 82.—To find the centre of curvature and focal length of a concave mirror.

If a small object is placed exactly at the centre of a concave mirror all the rays diverging from the object will meet the surface of the mirror normally, and, by the laws of reflexion, will be reflected back along their own paths, and will thus come to a focus again at the point from which they started. The distance of this point from the surface of the mirror is called the radius of curvature of the mirror. The focal length of the mirror is one-half its radius of curvature.

The concave mirror is set up vertically, and a single search-

pin is moved along its axis until the image of the pin formed by the mirror is seen in the same position as the pin itself. This is ascertained by moving the head from side to side, and adjusting the position of the pin, until the pin and its image coincide from whatever direction they are viewed. The image will be inverted and the greatest accuracy will be obtained if the height of the pin is adjusted so that the pin and its image are exactly point to point. The point of the pin will then be at the centre of curvature of the mirror. The distance of this point from the mirror can be measured accurately by taking a large pair of dividers and opening them until one point exactly touches the pin, and the other the centre of the mirror surface. The dividers are then placed on a metre scale and the distance read off to a fraction of a millimetre. This method is necessary with a concave mirror because, owing to the concavity of the surface, it is not possible to place the end of a scale in contact with the centre of the mirror surface. The method can also be used with advantage in other cases.

Three or four independent settings of the search-pin should be made and the mean taken. (This should be done in all cases when locating optical images either by the parallax or screen method.) The results may be recorded as follows :

DISTANCE OF CENTRE FROM MIRROR

	24.32 cm.
	24.38 „
	24.30 „

Mean	24.33 cm. = radius of curvature

Focal length of mirror = 12.17 cm.

EXPERIMENT 83.—To investigate the nature of the images formed by a concave mirror, and to verify the formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

The mirror is clamped in a vertical position, and a lighted burner with gauze in front is set up in front of the mirror and close to it. On looking into the mirror an upright and magnified image of the gauze is seen apparently behind the mirror. Since the image is behind the mirror it is obviously virtual. The burner is gradually removed from the mirror, and the image

is seen moving further behind the mirror and becoming larger and larger. Eventually a point will be reached when no image is seen but the whole mirror appears full of light. The gauze is then approximately at the focus of the mirror. If the burner is moved still further from the mirror no image will be seen behind the mirror, but if a cardboard screen is placed at some considerable distance in front of the mirror and then gradually brought nearer, a real image can be obtained on the screen. This image is magnified.

Continue to move the burner further from the mirror, moving the screen so as to keep the image focused upon it. As the object moves outwards the image moves inwards, until image and object coincide at the centre of curvature of the mirror, when both are of the same size. If the object is now moved still further from the mirror, the screen must be moved closer to the mirror in order to keep the image in focus. The image will now be smaller than the object. Draw up a table showing the position of the object (*e.g.* between the mirror and focus, between focus and centre of curvature, beyond the centre of curvature), and the nature, size, and position of the corresponding image. Draw diagrams showing how each of the images is formed.

It is shown in elementary text-books that if v is the distance of the image from the mirror, u the distance of the object from

the mirror, and f the focal length, then $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$. This formula

can be verified experimentally for real images by proceeding as in the previous part of the experiment and recording the distances of the object (*i.e.* the wire gauze) from the mirror, and the corresponding positions of the screen when the image of the gauze appears sharply focused upon it. The sum of the reciprocals of u and v should be constant. The values of

$\frac{1}{v}$ and $\frac{1}{u}$ can be obtained most conveniently from a table of reciprocals, taking care to get the decimal point in the right place. If the mirror used is the one employed for Experiment 82, the value of f obtained in this way should be equal to one-half the radius of curvature.

The experiment may now be repeated, using the parallax method of finding the image. A mounted pin is used as the object, a small piece of paper being placed on it so as to

distinguish it clearly from the search-pin. The object-pin is placed at different measured distances from the mirror, and the position of its image is obtained by moving the search-pin along the axis, until there is no parallax between the search-pin and the image of the object-pin (which can be distinguished from that of the search-pin, which will also be visible, by the piece of white paper). In both cases the results may be recorded as follows :

Distance of Object from Mirror = u .	Distance of Image from Mirror = v .	$\frac{1}{u}$	$\frac{1}{v}$	$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$
15.2 cm.	66.0 cm.	0.0660	0.0152	0.0812
18.5 "	36.3 "	0.0541	0.0276	0.0817
22.3 "	28.2 "	0.0450	0.0366	0.0816
27.6 "	22.2 "	0.0363	0.0451	0.0814
35.5 "	18.7 "	0.0282	0.0535	0.0817
42.8 "	17.2 "	0.0234	0.0582	0.0816
50.5 "	16.1 "	0.0198	0.0621	0.0819
Mean . . .				0.0816
$\therefore \frac{1}{f} = 0.0816 \quad \therefore f = 12.2 \text{ cm.}$				

The results may be expressed graphically by plotting the values of $\frac{1}{u}$ against the corresponding values of $\frac{1}{v}$ on squared paper. The points should all lie on a straight line, cutting both the axes at equal distances from the origin. The distance along either axis at which the line cuts the axis is equal to $\frac{1}{f}$.

EXPERIMENT 84.— To investigate the images formed by a convex lens, and to verify the formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Set up a convex lens in its holder, and proceed as in Experiment 83, placing the object *behind* the lens and looking at it *through* the lens. When the object is between the lens

and its focus the image is magnified, erect, and virtual. When the object is just beyond the focus the image will probably disappear. To recover it, stand back several feet from the lens and, looking in the direction of the object through the lens, fix your attention on the space between you and the lens. You will then catch sight of the image, which will be magnified, inverted, and real, and very much on your side of the lens. You can then focus it on a screen, or fix its exact position by the parallax method. Record the distance of image and object from the lens. Note that the former is negative. Move the object a little further from the lens, and again find the position of the image. It will be nearer the lens than before, and not quite so large. Obtain in this way a series of observations as in Experiment 83. Verify that $\frac{1}{v} - \frac{1}{u}$ is a constant for the lens (*a*) by plotting a graph between $\frac{1}{u}$ and $\frac{1}{v}$, (*b*) by direct calculation. From the formula this constant is equal to $\frac{1}{f}$. Calculate the mean value of f . Notice that it is negative. The results may be recorded as in the following example :

<i>u</i> cm.	<i>v</i> cm.	$\frac{1}{u}$	$\frac{1}{v}$	$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$
24.8	- 105	0.0403	- 0.0095	- 0.0498
26.2	- 84.5	0.0382	- 0.0118	- 0.0500
30.7	- 57.3	0.0326	- 0.0175	- 0.0501
35.6	- 45.9	0.0281	- 0.0218	- 0.0499
51.3	- 32.8	0.0195	- 0.0305	- 0.0500
Mean . . .				- 0.0500
$\frac{1}{f} = - 0.0500 \quad \therefore f = 20.0 \text{ cm.}$				

EXPERIMENT 85.—To verify the formula for a convex lens, using the virtual images.

This experiment is essentially less accurate than the preceding one, and would not be used in practice as a means of finding the focal length of the lens. It affords a useful exercise in locating virtual images. Use the same lens as in the preceding experiment, and place a small pin close behind the lens to serve as an object. Take a second much longer pin, sufficiently long to project well above and below the margin of the lens, and, looking at the object-pin through the lens and the search-pin above or below the lens, adjust the position of the latter until the search-pin appears to coincide without parallax with the image of the object-pin as seen through the lens. Several settings of the search-pin should be made, and the mean value of v determined. The experiment is then repeated with a different distance between the object-pin and the lens. Since the image is on the same side of the lens as the object, the value of v is positive. Record the results as follows:

u cm.	v cm.	$\frac{1}{u}$	$\frac{1}{v}$	$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$
6.1	8.6	0.1639	0.1163	- 0.0476
8.6	15.3	0.1163	0.0654	- 0.0509
10.4	21.4	0.0962	0.0467	- 0.0495
15.3	65.0	0.0654	0.0154	- 0.0500
Mean . . .				- 0.0495
$\therefore \frac{1}{f} = - 0.0495. \quad f = - 20.2 \text{ cm.}$				

EXPERIMENT 86.—To determine the focal length of a convex lens.

(a) The focal length of a convex lens can be determined by the method of Experiment 84, taking at least three independent positions of object and image. The measurements will be found to be most accurate when u and v are nearly equal.

The following method is, however, more rapid and less liable to error, as it involves no calculation.

(b) A plane mirror is placed behind the convex lens (Fig. 56) so that its plane is at right angles to the axis of the lens. A single pin is placed in front of the lens, and moved backwards or forwards until an image of the pin is seen, which is made to coincide with the pin itself without parallax. The pin is then at the focus of the lens, and the distance of the pin from the lens may be measured accurately by means of a pair of dividers and a scale. If the thickness of the lens is appreciable in comparison with the distance of the pin from the lens, the thickness of the lens may be measured by a pair of sliding calipers, and half the thickness of the lens added to the measured distance. This will give the distance of the focus from the

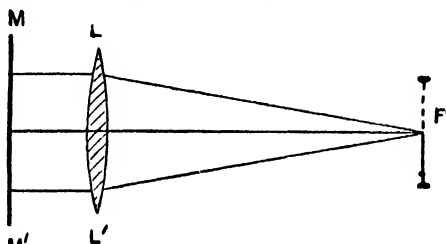


FIG. 56.—Determination of the Focal Length of a Convex Lens by use of a Plane Mirror.

centre of the lens. The setting of the pin should be made several times, and each measurement recorded. The mean of these independent values, which should not differ among themselves by more than a millimetre, should be taken.

In order that the rays from the tip of the pin should return to their starting-point, they must all have struck the plane mirror normally—that is to say, they must form a parallel beam after passing through the lens. But to do this they must emerge from the focus of the lens. Hence, when the image and object coincide, the object is at the focus of the lens.

The results may be recorded as in the following example :

Distance of object from nearer surface of lens for coincidence
 $= 20.03 \text{ cm.}, 20.05 \text{ cm.}, 20.04 \text{ cm.}$ Mean $= 20.04 \text{ cm.}$

Thickness of lens $= 0.22 \text{ cm.}$

Focal length of lens (distance of focus from centre of lens)
 $= 20.04 + 0.11 = 20.15 \text{ cm.}$

(c) A very accurate method of determining the focal length of a convex lens consists in finding the *minimum* distance between an object and the real image of it formed by the lens. It can be shown that the distance between object and image is least when each is at the same distance from the lens, and it follows from this that the minimum distance is exactly four times the focal length of the lens. The student may verify this by putting $v = -u$ in the formula. The distance between the object-pin and the search-pin can be determined much more accurately than the distance of either from the lens, and the result is, therefore, less liable to error.

Place the object-pin at rather more than twice the focal length from the lens, and find the image in the usual way. Now move the lens slightly towards the object, and again find the image. If the search-pin has to be moved towards the object-pin, the lens is being moved in the right direction. Continue the process until the distance between the two pins is a minimum, when moving the lens still further in the same direction will cause the image to move in the opposite direction. When the minimum distance has been found, remove the lens and measure the distance between the pins with a metre scale. As the pin-points form very good objects to measure from, the reading may be made to one-tenth mm. One-quarter of this distance is the focal length of the lens. This method takes longer to carry out than the mirror method, but is more accurate.

EXPERIMENT 87.—To measure the magnification produced by a convex lens.

Take as the object either a piece of wire gauze or a circular hole in a screen illuminated from behind by a luminous flame. Place the object at such a distance from the lens that a real image is formed, and focus it sharply upon a white screen. Measure the distance between ten threads of the gauze (*a*) on the object itself, (*b*) on the image on the screen, using a pair of dividers. If the circular hole is used, measure the diameter of the hole itself and the diameter of its image in the same way. The ratio of the diameter of the image to the diameter of the object is the magnification produced by the lens. Measure the distances of the object and image from the lens and show that

$$\text{Magnification} = \frac{\text{size of image}}{\text{size of object}} = \frac{v}{u}$$

mirror. The plane mirror is then moved in the line of sight until there is no parallax between these two images. The conditions are then as shown in Fig. 57.

The image I formed by the plane mirror is as far behind the mirror M as the object is in front. But since there is no parallax between the two images, I is also the position of the image formed by the convex mirror. Hence for the convex mirror, $CO = u$, and $CI = v$. But $CI = MI - MC = MO - MC = OC - 2MC$. The distance MC is, of course, measured from the back or silvered surface of the plane mirror. By taking different positions of the object O , the formula can be verified. It should be remembered that v is negative.

EXPERIMENT 89.—To verify the formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for a concave lens.

This experiment may be carried out either by the method of Experiment 85 or by a modification of the method just

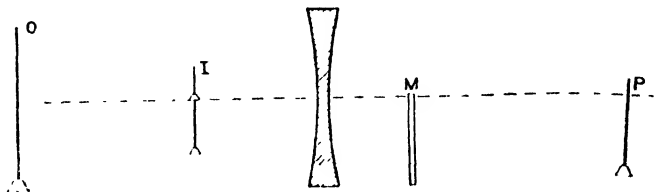


FIG. 58.—Location of the Image formed by a Concave Lens, using a Plane Mirror.

described. A pin O (Fig. 58), is placed behind the concave lens to serve as an object, and a plane mirror M is placed in front of the lens and half covering it. A second pin P is moved about in front of the mirror until the image of this pin seen in the mirror coincides without parallax with the image of the pin O seen through the upper half of the lens. This image is thus situated at I , where $IM = MP$.

It will be found that the two preceding experiments do not yield very accurate or consistent results. To determine the focal length of either a convex mirror or a concave lens accurately we require auxiliary apparatus which will enable us to work entirely with real images. A convex lens will answer our purpose.

EXPERIMENT 90.—To determine the radius of curvature and focal length of a convex mirror.

A convex lens is placed on its stand and a pin P (Fig. 59) is placed just beyond its focus so that a real image is formed at some point C beyond the lens. The position of C is found by the method of parallax, or if preferred a luminous object and a focusing screen can be used instead. The distance CB of the image from the lens is then measured. The convex mirror is then inserted between the lens and C, and *without moving either the lens or the object-pin* the mirror is moved in the line of sight until an inverted image of the pin is seen coinciding with the pin itself. The rays from the pin must after passing through the lens have met the surface of the mirror normally, as they have been made to retrace their

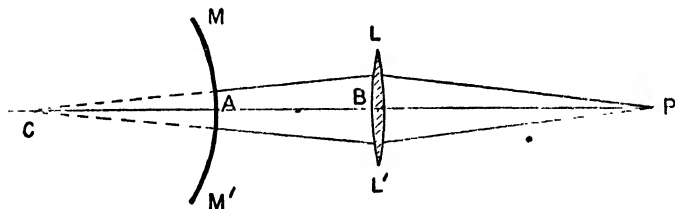


FIG. 59 —Determination of the Radius of Curvature of a Convex Mirror.

paths, *i.e.* they must have been travelling along radii of the mirror. Hence the point C to which they were converging must be the centre of the convex mirror. The distance AB between mirror and lens is measured by means of dividers. $BC - AB$ is obviously the radius of curvature of the mirror. The focal length of a mirror is one-half its radius of curvature.

EXPERIMENT 91.—To determine the focal length of a concave lens. (First method.)

A convex lens of short focal length is selected and placed in contact with the concave lens. The combination is then tested to make sure that it acts as a convex lens. This may be done by holding it above a page of print and seeing if magnification is produced. The focal length of the combination can then be determined as in Experiment 86. It will probably be fairly large. The focal length of the single convex lens is determined in the same way. If F is the focal

length of the combination of lenses, f_1 that of the single convex lens, and f_2 that of the concave lens

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

and f_2 can then be calculated. Remember that both F and f_1 are *negative* in sign. Thus if the focal length of the combination is 45 cm., and that of the single convex lens 10 cm., we have

$$-\frac{1}{45} = -\frac{1}{10} + \frac{1}{f_2}; \quad \frac{1}{f_2} = \frac{1}{10} - \frac{1}{45}$$

$$f_2 = 12.9 \text{ cm.}$$

EXPERIMENT 92.—To determine the focal length of a concave lens. (Second method.)

The concave lens is mounted in its stand and a convex lens (which need not be of shorter focal length) is placed a few centimetres in front of it as shown in Fig. 60. A pin A is

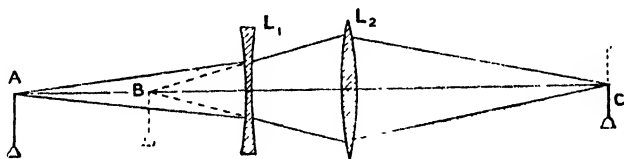


FIG. 60.—Determination of the Focal Length of a Concave Lens.

placed some distance behind the concave lens and moved about until a real inverted image appears in front of the convex lens. The position of this is located, say at C, in the usual way. The distance AL_1 of the pin A from the concave lens is measured, also the distance L_1L_2 between the two lenses, and the concave lens is removed. The pin A is moved towards the convex lens until its image again coincides with the pin at C (which must not have been moved). If B is the second position of the pin, then since the rays from B are brought to a focus at C by the convex lens, B must be the point from which the rays from A appeared to be diverging after refraction through the lens L_1 . That is to say, B is the image of A formed by the concave lens. The distance BL_2 is measured, and L_1L_2 subtracted from it. This gives BL_1 or v . The distance $AL_1 = u$ has already been measured.

The focal length can be calculated from the formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

§ 26. MEASUREMENT OF REFRACTIVE INDICES

ONE method of measuring the refractive index of a substance is described in Experiment 81. It is not capable of sufficient accuracy to be of much practical value. The accurate determination of refractive indices involves the use of elaborate apparatus. The following experiments will illustrate some of the methods employed, and at the same time will, with care, give an accuracy of within 1 or 2 per cent.

It is well known that an object when viewed through a refracting medium appears to be closer to the eye than its real distance. Thus a pond always appears shallower than it really is. It can be shown that if we are looking vertically down on the surface, that is normally to the surface, the ratio of the real depth to the apparent depth of the pond is equal to the refractive index of water. This is true for any refracting medium. Hence if we can determine the apparent position of an object seen through the plane surface of a refracting substance we can determine the refractive index. Either of the following methods may be employed :

EXPERIMENT 93.—To determine the refractive index of water by measuring the apparent depth.

A tall gas jar is filled nearly to the brim with water, and a pin is dropped to the bottom of the jar to serve as an object. The distance from the pin to the surface of the water is measured by a metre scale and gives the true depth of the water. To find the apparent depth place a strip of good plane mirror glass across the top of the jar so that the edge of the mirror is at right angles to the pin. Clamp a pin in a tall retort stand so that the image of this pin seen in the mirror is in the same straight line as the image of the lower pin seen through the water. Adjust the height of the pin in the clamp until there is no parallax between these two virtual images. The two images are then as far below the surface of the

mirror as the pin in the clamp is above it. Measure this distance. Subtract from it the distance between the surface of the mirror and the surface of the water, the measurements being taken from the silvered surface of the mirror. The difference is obviously the apparent depth of the lower pin below the surface of the water. The adjustment should be made several times, as it is not particularly easy to obtain the exact position of coincidence, and the mean taken. The refractive index of water can then be calculated as follows :

$$\begin{aligned}
 \text{Real depth of water} &= 22.5 \text{ cm.} \\
 \text{Distance of pin above plane mirror} &= 19.4, 19.2, 19.5 \text{ cm.} \\
 &\quad \text{Mean distance} = 19.4 \text{ cm.} \\
 \text{Distance between mirror and surface of water} &= 2.4 \text{ ,,} \\
 \text{Apparent depth of water} &= 19.4 - 2.4 \text{ cm.} = 17.0 \text{ ,,} \\
 \text{Refractive index of water} &= \frac{\text{real depth}}{\text{apparent depth}} = 1.32
 \end{aligned}$$

If the upper pin is brightly illuminated it is possible to see its reflection in the upper surface of the water itself. The glass mirror can then be dispensed with, and the reflection in the water used instead. The distance of the upper pin from the surface of the water will then be equal to the apparent depth. The image formed by the water surface is more perfect than that formed by the average glass mirror, and will thus give more accurate results. Owing to its faintness it will not be found so easy to work with until some practice has been obtained.

Another method of locating the image is given in the following experiment :

EXPERIMENT 94.—To determine the refractive index of a block of glass by measuring its apparent thickness.

A vertical mark is made on the back surface of a thick, parallel-sided block of glass by sticking a straight-edged gummed label to it, and the block is placed on the table. A convex lens is placed in front of the block (Fig. 61), so that the axis of the lens is normal to the surface of the block, and is moved about until a real inverted image of the mark is formed. The position of this image is then located by the method of parallax. The distance between the nearer surfaces of the lens and the block is then measured, using a pair of dividers. Without moving the search-pin or the lens the

block is removed, and a second pin is adjusted on the side of the lens where the block had been until the image of the first search-pin, as seen through the lens, coincides with the second pin without parallax. The second pin now occupies the same

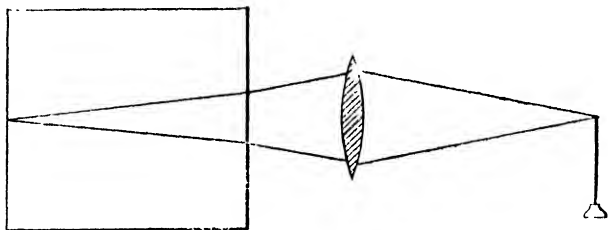


FIG. 61.—Determination of the Apparent Thickness of a Glass Block, using a Convex Lens.

position as the virtual image of the mark on the block. Measure its distance from the lens. Subtract the measured distance of the front surface of the block from the lens. The difference is the apparent thickness of the block. The true thickness can be measured with a pair of sliding calipers.

The ratio of the real to the apparent thickness is the refractive index.

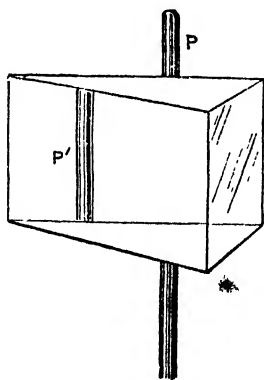


FIG. 62.—Showing the Deviation produced by a Prism.

The refractive index of a substance which can be made into a triangular prism is best determined by measuring the deviation which the prism produces in the path of a ray of light passing through it. The deviation is best measured by an accurate instrument known as a spectrometer, but it can be determined in an elementary laboratory by tracing the path of a ray of light through the prism by means of pins.

The deviation produced depends on the angle which the incident ray makes with the surface of the prism. Look at some vertical object such as a retort stand through a glass prism. The portion of the retort stand seen through the prism will

appear to be displaced towards the refracting edge of the prism as at P^1 (Fig. 62). If the prism is now slowly rotated about the refracting edge the image P^1 will be seen to move either nearer or further away from the position of the object. If we turn the prism in such a direction that the image and object move closer together, we shall find, on continuing the turning in the same direction, that a point is reached where the image first remains stationary, and then begins to move further away from the line of the object. The position where the displacement of the image is least is called the position of minimum deviation, and it is the minimum deviation which is always measured.

It can be shown that if δ is the angle of minimum deviation (Fig. 63), that is to say, the minimum angle between the

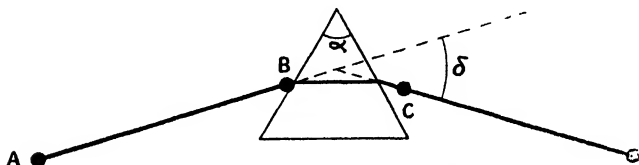


FIG. 63.—Measurement of the Deviation produced by a Prism.

incident and emergent rays, and α is the angle of the prism, that is, the angle between the two refracting faces, the refractive index μ of the material is given by

$$\mu = \frac{\sin \frac{\alpha + \delta}{2}}{\sin \frac{\alpha}{2}}$$

The angle of minimum deviation can be determined as follows:

EXPERIMENT 95.—To measure the angle of minimum deviation of a prism.

Place the prism (e.g. a 60° glass prism) on a drawing-board covered with white paper, and on one side of the prism set up two pins A and B (Fig. 63) to indicate the direction of an incident ray. AB may conveniently make an angle of about 60° , with the normal to the surface of incidence. Looking

from the other side of the prism, place two more pins C and D so that all four pins appear on the same straight line when A and B are viewed through the prism. CD is then the line of the emergent ray, and the angle between CD and AB produced is the angle of deviation δ . Rotate the prism through a few degrees so as to decrease the angle of incidence, and again trace the emergent ray. It should make a somewhat smaller angle with the incident ray than before. If not, the prism must be rotated in the opposite direction. Continue until the position of minimum deviation is reached. Draw a line round the edges of the prism while in this position, then measure by means of a large protractor the angle α of the prism and the angle of minimum deviation. Substitute these values in the equation, and determine the refractive index of the material of the prism.

Example :

$$\begin{array}{ll} \text{Angle of prism} & = 60^{\circ}0 \\ \text{Angle of minimum} & \\ \text{deviation} & = 39^{\circ}5 \end{array}$$

$$\text{Refractive index} = \frac{\sin \frac{60^{\circ} + 39^{\circ}5}{2}}{\sin \frac{60^{\circ}}{2}} = \frac{\sin 49^{\circ}25'}{\sin 30^{\circ}} = \frac{0.7615}{0.5000} = 1.523.$$

EXPERIMENT 96.—To make a simple spectrometer.

Take two straight pieces of wood about 20 cm. long and 5 cm. wide, with straight edges, and to one end of each fasten a convex lens of about 15 cm. focal length. On one of the boards place a pin on a small stand. Place a plane mirror behind the corresponding lens and move the pin until it coincides with its inverted image, as seen in the lens and mirror. The pin is now at the focus of the lens. Secure it in this position with soft wax.

Place the two boards in line with the convex lenses together, and mount a small screen with a vertical slit on the further end of the second board. Illuminate the slit from behind by a flame, and slide it backwards or forwards until the image of the slit is focused on the pin, without parallax. The slit is then at the focus of the other lens, and the light radiating from the slit forms a parallel beam of light between the two lenses. The lens and slit form the collimator of the simple spectrometer, the other lens and pin form the telescope. It is

convenient to have the pin and the slit mounted on small slides which move in grooves cut along the centres of the two boards, as shown in Fig. 64, but this is not essential. They can be secured in their proper positions by a little soft wax or sealing wax.

In an actual spectrometer, the telescope and collimator are hinged, so as to rotate about the same vertical axis. A second small magnifying-glass is used to view the pin, and engraved brass scales fitted with verniers are used for reading the position of the two arms. These devices add materially to the con-

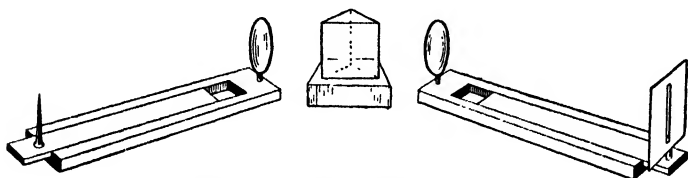


FIG. 64.—A Simple Spectrometer.

venience and accuracy of the instrument, but the principle is the same as that of our simple instrument.

EXPERIMENT 97.—To measure the angle of minimum deviation, and the angle of a prism, by the simple spectrometer.

The spectrometer is adjusted as described in the previous experiment, and the telescope and collimator are placed on a sheet of drawing-paper on a drawing-board. The two lenses are placed facing each other, but with sufficient space between them to leave room for the insertion of the prism. The telescope is then moved until the image of the slit is exactly on the pin. The telescope and collimator are then in line. A pencil line is ruled down one edge of the telescope. The prism is then inserted between the collimator and the telescope, and the latter is turned until the image of the slit is again on the pin. The prism is then slowly rotated, the telescope being moved so as to keep the image still on the pin, until the position of minimum deviation is reached. This position can be identified by the fact that after this point the image of the slit begins to retrace its path.

Some little difficulty may be experienced in fixing the exact position of the telescope if the slit is illuminated with white light, because it will be found that instead of an exact image

of the slit a narrow band of bright colours is formed in the position of the pin. This is due to the dispersion of the white light in the prism. The refractive index of glass for blue light is rather greater than for red light, and the red components of the white light are thus deviated rather less by the prism than the blue. The effect is very well seen if a small magnifying-glass is used to view the pin and the image. To obviate this difficulty, we may illuminate the slit with monochromatic light, by placing a piece of red glass over the slit.

Having obtained the exact position of minimum deviation, a pencil line is again ruled along the same edge of the telescope

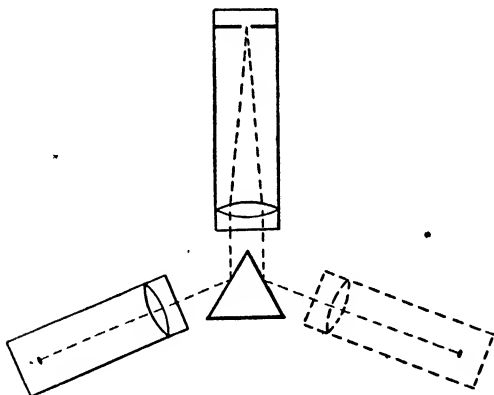


FIG. 65.—Measurement of the Angle of a Prism by the Simple Spectrometer.

as before. The angle between these two lines is the angle through which the telescope has been turned, and, obviously, measures the deviation of the ray by the prism. This method of procedure eliminates any error which might otherwise arise from the optical axis of the telescope not being exactly parallel to the edge of the board on which it is mounted.

To measure the angle of the prism the prism is placed so that its refracting edge is facing the lens of the collimator (Fig. 65). The telescope is turned to face one face of the prism and is adjusted until an image of the slit reflected from this face of the prism is formed on the pin. A line is then ruled along the edge of the telescope. The operation is repeated with the other face of the prism, the collimator and

prism remaining in exactly the same positions as before. The parallel beam of light from the collimator falls partly on one face of the prism and partly on the other. The angle between the two directions in which it is reflected from the two surfaces, that is, the angle between the two positions of the telescope, is thus equal to twice the angle between the two faces.

The angles may be measured with a large protractor, fractions of a degree being estimated. The refractive index of the prism can then be calculated as in Experiment 95.

If a ruby glass has been placed before the slit, this will give the refractive index for red light. By substituting a blue glass the refractive index for blue light can be measured in the same way. The difference in the two indices will be small (it is rather less than 1 per cent. for ordinary glass), but it should be appreciable with the apparatus described, if care is taken in setting the telescope.

§ 27. SOUND

THE FREQUENCY OF A TUNING-FORK

SOUND is caused by vibration, and is propagated through the air by wave motion. The prongs of a tuning-fork when it is sounding are obviously in rapid motion though the individual vibrations are too rapid to be followed by the eye. If, however, a short bristle is fastened to one prong of a tuning-fork, and the latter, after being struck to make it sound, is drawn rapidly across the surface of a blackened plate, so that the bristle is just in contact with the surface, a wavy line is left on the plate, each complete wave corresponding to one complete vibration of the prong. If the time taken in drawing the fork across the plate could be accurately measured, the number of complete waves on the plate divided by the time would give the number of vibrations made by the fork per second, that is, the frequency of the fork. The determination of frequency can actually be made on this principle, using a smoked drum driven at a known number of revolutions per second. We can easily compare the frequencies of two forks by this method without any elaborate apparatus.

EXPERIMENT 98.—To compare the frequencies of two tuning-forks.

Attach a short bristle to one prong of each fork by means of a little soft wax. Use as little wax as possible, as loading the prong of a tuning-fork reduces its frequency. Blacken a glass plate by holding it over a smoky lamp flame (burning camphor is better), and place it blackened side uppermost on the bench. Strike both forks on a block of wood, or a large rubber cork, to make them vibrate, and then, holding both the forks close together in one hand draw them quickly over the plate, so that the bristles are just in contact with the plate. Each bristle will then trace a wavy line on the plate. Draw

two parallel lines about 10 cm. apart at right angles to the direction of the waves, and count the number of complete waves included between these two parallels for each wave trace. Since both forks were moved across the plate at the same speed, these numbers are the numbers of vibrations made by the two forks in the same time, that is to say, they are proportional to the frequencies of the forks.

If the frequencies of the two forks are very nearly the same the difference between them can be found by the method of beats. When the two forks are sounding simultaneously the sound heard will have rhythmic variations in intensity, and will apparently reach the ear in a regular succession of pulses or "beats" with intervals of comparative silence between. It can be shown that the number of beats per second is equal to the difference in frequency of the forks. If the beats are sufficiently slow to be counted (that is, in practice if there are not more than 10 per second), the difference in frequency can be determined.

EXPERIMENT 99.—To determine the difference in frequency between two tuning forks by the method of beats.

Two tuning-forks of nearly equal frequency will be required, and we will suppose that the frequency of one of them is known. Sound the two forks together, and press the stems on a sounding-board, or failing this on the bench. Count the number of beats heard in 10 seconds, if the forks will vibrate for as long as this. Remember that the first beat must be counted "nought" and not "one" (see page 39). Repeat the observations three or four times, and find the average number of beats per second. This is the difference in frequency of the forks.

To find which of the forks has the greater frequency, load the prongs of the unmarked fork with soft wax, and again count the beats. The wax will reduce the frequency of the fork. Now if the frequency of the unmarked fork was already less than that of the marked fork the loading will increase the difference between them and hence the beats will be more rapid. If, however, the unmarked fork was of higher frequency the loading will making the frequencies more nearly equal and the number of beats per second will be less.

Example :

Standard fork, frequency = 256 per second.

Number of beats in 10 seconds when sounding with unmarked fork X = 43, 42, 42.

Number of beats per second = 4.2.

Unmarked fork when loaded with wax made 10 beats in 5 seconds.

Hence unmarked fork has a greater frequency than standard.

Frequency of fork X = $256 + 4.2 = 260.2$ per second.

§ 28. EXPERIMENTS ON STRETCHED STRINGS

IF a flexible elastic string is tightly stretched and plucked or struck it will vibrate, giving out, if the vibrations are of suitable frequency, an audible note. The sound will be much louder if the string is stretched over some kind of sounding-board. The frequency of the fundamental vibration of the string (that is, when the string is vibrating as a whole and not in separate segments) is given by

$$n = \frac{1}{2l} \sqrt{\frac{F}{m}}$$

where l is the length of the vibrating portion of string, F the stretching force (in dynes), and m the mass of unit length

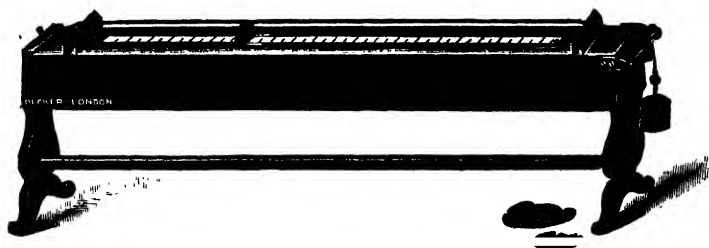


FIG. 66.—The Sonometer.

of the string. The material of the string does not affect the frequency (except as it affects m), and the formula may be applied either to strings or to wires.

The formula can be tested experimentally by means of an instrument known as a monochord or sonometer (Fig. 66). This consists of a long hollow wooden box (or a long wooden board mounted on strips of wood at each end) to serve as a sounding-board. At one end are one or more stout pegs to which the string or wire can be attached: a thin pianoforte steel wire is convenient. At the other end is a pulley over which the string passes so that tension may be applied by

hanging known weights upon it. A scale pan is usually attached to the wire. Two metal bridges, one of which is movable, are placed under the string at various points along its length to adjust the length of the vibrating segment. A metre scale screwed to the surface of the instrument enables the distance between the bridges to be measured. It will be well to ascertain from some one in authority before commencing the experiment what is the maximum load which the string or wire is capable of carrying, and to keep the load well within the safety limit. Unpleasant accidents may be caused by the sudden snapping of a long, heavily loaded wire.

EXPERIMENT 100.—To show that the frequency of a stretched string is inversely proportional to the length.

The sonometer is set up with the bridges at opposite ends of the string, and weights are added to the pan until the string, on being plucked in the middle, emits a note in unison with a standard fork making, say, 256 vibrations per second. To a musical ear this will present no difficulties. Many students, however, find some difficulty in distinguishing between a note and its octave, and some find it impossible to tune the wire at all. When the experimenter has found the point at which the notes appear to him to be in unison, one of the bridges should be slightly displaced, first so as to make the length of the vibrating wire shorter, then so as to make it slightly longer. If in the first case the note appears distinctly higher in pitch, and in the second distinctly lower than the tuning-fork, the position found is probably nearly correct.

The exact position can then be found by the method of beats. The tuning-fork and the wire are sounded together, and the beats are listened for. The bridge is then moved very slightly. If beating becomes less rapid the frequency of the wire is approaching more nearly to equality with that of the fork. The bridge is then again adjusted until the beating becomes so slow that it cannot be detected. Another test which may be applied is that of sympathetic vibration. If the wire is in unison with the tuning-fork, and a small paper rider is placed lightly on the centre of the string, it will be thrown off if the tuning-fork is sounded and placed with its stem in contact with the sounding-board of the sonometer. The wire is set in vibration by the vibrations of the fork. This is known as sympathetic vibration or resonance. If the

bridge is moved slightly so that the wire is no longer in unison with the fork the sympathetic vibrations do not occur. Unfortunately both beats and sympathetic vibrations may arise between a note and its octave, so that neither method entirely eliminates the possibility already alluded to that we may be tuning our string in the wrong octave.

Having adjusted the string to unison with the standard fork, measure the distance between the two bridges. Keeping the load in the pan constant, select another tuning-fork of higher frequency, and adjust the distance between the bridges until unison is again obtained. Make observations with as many different tuning-forks as may be available. Plot a curve on graph paper between the frequency, as stamped on the fork, and the reciprocal of the distance between the bridges. The graph should be a straight line.

EXPERIMENT 101.—To show that the frequency of a stretched string is proportional to the square root of the stretching force.

This experiment may be carried out in a manner similar to that of the previous experiment. The bridges are fixed at a definite distance apart, say 50 cm., and the wire is tuned to unison with a given fork by adding weights to the scale pan.

The total weight of the scale pan and its contents is then the stretching force. The experiment is repeated with tuning-forks of other known frequencies. A graph is then drawn between the frequency of the fork and the square root of the weight of the pan and its contents.

This method is not very convenient. It involves the use of a large number of weights, and the adjustment of the frequency by adding or subtracting weights from the pan is somewhat tedious. We can improve the experiment by using the result obtained in the previous experiment that the frequency for a given stretching force is inversely proportional to the length of the vibrating wire. The length of wire required to give a definite constant frequency should thus be directly proportional to the square root of the stretching force.

Take a single tuning-fork (frequency about 256 per second), and place, say, 2 kgrms. in the pan. (The exact weights to be used will depend on the strength of the wire.) Tune the wire to unison with the fork by altering the position of the bridges as in the previous experiment. Record the distance apart of

the bridges and the stretching force (= weights in pan + weight of scale pan). The initial distance between the bridges should not be more than half the length of the wire. Now increase the weight in the pan by adding, say, another 2 kgrms., and again tune the wire to the same fork by moving the bridges. The bridges will now be further apart. Continue in this way until either the maximum safe load is reached or until the bridges can be moved no further apart. Record the measurements as in the following record, and plot a graph between the length of the vibrating wire and the square root of the stretching force.

Weight of scale pan = 200 gm. Frequency of fork = 256.			
Stretching Force = F .	Distance between Bridges = l .	\sqrt{F} .	$\frac{\sqrt{F}}{l}$
2200 gm. wt.	31.5 cm.	46.9	1.49
4200 " "	43.3 "	64.8	1.50
6200 " "	53.0 "	78.7	1.48
8200 " "	60.8 "	90.6	1.49

EXPERIMENT 102.—To measure the frequency of a tuning-fork by means of the sonometer.

Load the wire of the sonometer, and tune it to unison with the fork as already described. Several independent settings should be made, preferably with different loads in the pan, and the mean value of the ratio $\frac{\sqrt{F}}{l}$ determined as in the previous experiment. The wire is then removed from the sonometer, and a length of about 100 cm. is cut off and accurately measured. This length of wire is then weighed, and the value of m , the mass per unit length of the wire, determined. By substituting in the formula the frequency of the note emitted by the wire, and hence that of the tuning-fork can be determined. The stretching force must be converted into absolute units of force (dynes) by multiplying the weights by g , the acceleration due to gravity.

§ 29. THE RESONANCE TUBE

IF we blow across the edge of a tube containing air, we can make it emit a note of definite pitch, the frequency of which depends on the length of the tube. The note is generally mixed with a considerable amount of noise, but it is usually possible to recognise the pitch of the musical note. If now a vibrating tuning-fork, of the same pitch, is held over the open end of the tube, the column of air will be set into sympathetic vibration, and the faint note of the fork will be greatly enhanced. This effect is known as resonance.

It can be shown that if the tube is closed at one end, the length of the tube is equal to one-quarter of the wave length of the note emitted. If the tube is open at both ends, the length of the tube is equal to one-half the wave length. There is, however, in each case a small correction to be made. The column of air set in motion is somewhat longer than the actual tube, since the air will vibrate for a short distance beyond the end of the tube. The correction for the end effect amounts to 0.3 times the internal diameter of the tube. Hence, for a tube closed at one end the wave length λ of the note emitted is given by

$$\lambda = 4(L + 0.3 D),$$

where L is the length of the tube and D its diameter. For a tube open at both ends the formula is

$$\lambda = 2(L + 0.6 D).$$

If V is the velocity of sound in air at the temperature of the experiment, n the frequency of a note, and λ its wave length in air, it can be shown that

$$V = n\lambda.$$

Hence, if we measure λ by means of the resonance tube, we can calculate the frequency n of the note, if the velocity of sound in air is known, or conversely we can calculate the

velocity of sound in air, if the frequency of the note is given. The experiment is exactly the same in each case, and consists in determining the wave length of the note. It is only the calculation which is different.

EXPERIMENT 103.—To determine the wave length of a note in air by the resonance tube.

A simple form of resonance tube consists of a piece of glass tube, about 3 cm. in diameter, which is held in an upright position with its lower end dipping under water, contained in a tall glass jar (Fig. 67). The length of the vibrating column of air in the tube is increased by raising the tube, and decreased by lowering it still further into the water. This forms what is known as a *closed tube*, *i.e.* closed at one end. A more elaborate form consists of a pair of long glass tubes, mounted on a vertical stand, and connected at their lower ends by a long rubber tube. The lower half of the apparatus is filled with water, and the level of the water in the fixed tube can be adjusted by raising or lowering the other tube, which is mounted on a slide. This form is rather more convenient than the simpler kind, but is not essentially more accurate. We shall assume in the description which follows that the simple form is being used.

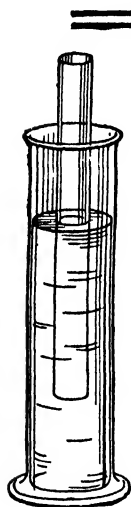


FIG. 67.—A Resonance Tube.

The tube is lowered well into the glass jar, so that the water rises nearly to the top of the resonance tube. A tuning-fork is then sounded, and held some two or three centimetres above the open end of the tube, which is at the same time gradually raised out of the water. As the vibrations of the fork die away, they must be renewed by striking it again. Eventually a point will be reached when the note of the fork, which has been only just audible, suddenly swells out. This is approximately the position of resonance. The length of the air column is now carefully adjusted until the intensity of the sound is a maximum. The distance from the top of the resonance tube to the level of the water is measured. The experiment is repeated two or three times, and the mean taken. The dia-

meter of the tube is also measured ($=D$). The wave length of the note is then equal to $4(L+0.3D)$.

If the tube is sufficiently long, a second position of resonance can be found when the length of the air column is approximately three times the length for the first point of resonance. The difference between these two lengths is exactly equal to one-half the wave length of the note, no end correction being required. This is the most accurate way of determining the wave length, if the tube is sufficiently long to give the second point. For a tuning-fork of frequency 256, the length of tube required would be about 1 metre. A tube just over half a metre long would enable the second point to be determined, using a tuning-fork of frequency 512.

The wave length λ having been determined, we can then use the equation $V=n\lambda$ to determine either (a) the velocity of sound in air if the frequency of the tuning-fork is given, or (b) the frequency of the fork if the velocity of sound is given at the temperature of the experiment.

Example :

Temperature of experiment $= 16^{\circ} \text{C.}$

Length of air column for first point of resonance

$= 21.6, 21.4, 21.8 \text{ cm.}$

Mean $= 21.6 \text{ cm.}$

Length of column for second point of resonance

$= 65.7, 66.0, 65.9 \text{ cm.}$

Mean $= 65.9 \text{ cm.}$

Diameter of tube $= 2.0 \text{ cm.}$

Wave length of note (from first observation alone)

$= 4(21.6 + 0.3 \times 2.0) = 88.8 \text{ cm.}$

Wave length of note from two positions

$= 2(65.9 - 21.6) = 88.6 \text{ cm.}$

Frequency of tuning-fork (given)

$= 384 \text{ per sec.}$

Hence velocity of sound in air at 16°C.

$= 384 \times 88.6 \text{ cm. per sec.}$

$= 340 \text{ metres per sec.}$

ADDITIONAL EXERCISES AND EXAMINATION QUESTIONS.—IV

1. Place a plane mirror vertically and draw a line ABC along the edge of the mirror and another BD at right angles to ABC. Place pins on BD to mark the position of a ray of light. Move the mirror about a vertical axis through B through a series of different angles and find for each position the direction of the reflected ray. Draw a graph between the angle turned through by the mirror and the angle turned

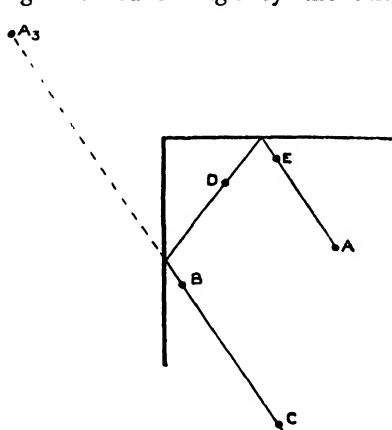


FIG. 68.—Path of a Ray of Light between Two Mirrors at Right Angles.

through by the reflected ray, and deduce the relation between them.

2. Place two plane mirrors at right angles to each other. Find the positions of the three images formed and trace the paths of the rays by which the images are seen.

(The paths of the rays may be traced by pins as in Experiment 77. The image in the angle behind the mirrors is formed by a ray which has been successively

reflected at both mirrors. A (Fig. 68) is the object-pin. The pins B and C are then placed in line with the image A, and then the pins D and E. AEDCB is then the path of a ray. A second ray can be traced in the same way.)

3. Make experiments with the given parallel-sided block of glass to show that a ray of light entering one face will emerge in a parallel direction from the opposite face.

4. Plot a curve showing how the distance between an object and its real image formed by a convex lens varies with the distance of the object from the lens. Find the least distance between image and object and deduce the focal length of the lens.

5. Arrange a convex lens and two scales so that the image

ADDITIONAL EXERCISES AND QUESTIONS 177

of one scale is formed without parallax on the other and is magnified by two. Measure the distance between the scales. Deduce the focal length of the lens.

(A glass scale should be used for locating the image. If I and O are the dimensions of image and object $\frac{v}{u} = \frac{I}{O}$ and $v + u =$ distance between scales. Hence v and u can be determined.)

6. Find the focal length of the given concave lens. Determine the radii of curvature of its faces by using them as concave mirrors and deduce the refractive index of the material of the lens from the formula

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{r} + \frac{1}{s} \right)$$

where r and s are the numerical values of the two radii, and μ is the refractive index.

(In determining the radii of curvature a black cloth should be placed behind the lens, and a brightly illuminated pin in front. The real image of this pin formed by the concave surface of the lens will then be clearly visible, and its position can be found by parallax.)

7. Place the given concave mirror with its axis vertical. Adjust a pin so that it coincides with its reflected image. Fill the mirror with water and repeat the observations. From the distances of the pin from the position of the water surface deduce the refractive index of water. (If d_1 , d_2 are the distances of the pin from the position of the water surface in the two cases, it can easily be shown that $\mu = \frac{d_1}{d_2}$. The distances

should be measured from the mirror surface in the first place and the depth of water subtracted from each measurement.)

8. Place the given convex lens on a horizontal plane mirror, and determine its focal length. Interpose between the lens and mirror (a) a little water, (b) a few drops of the given liquid (e.g. aniline). Determine in each case the focal length of the combination of the convex lens and the concave liquid lens, and deduce that of the liquid lens. Hence deduce the refractive index of the liquid, that of water being 1.33.

(The liquid lens is bounded by the plane mirror and the surface of the convex lens. Its radii of curvature are thus

identical for both liquids and hence by the formula in Question 6

$$\frac{\mu_w - 1}{\mu_a - 1} = \frac{f_a}{f_w}$$

where f_a and f_w are the focal lengths of the aniline and water lenses.)

9. Adjust the length of the given tube for resonance with tuning-forks of different known frequencies. Plot a graph between the reciprocal of the frequency and the corresponding length of the resonance column, and deduce from your graph a value for the "end correction" for the tube.

BOOK V

MAGNETISM AND ELECTRICITY

§ 30. QUALITATIVE EXPERIMENTS IN MAGNETISM

IF a bar-magnet is suspended so that it hangs horizontally and is free to turn in a horizontal plane, it will set itself in a definite direction, which is approximately north and south. If displaced from this position it will oscillate for some time, the oscillations getting smaller and smaller, and will finally come to rest in exactly the same position as before. The best way of suspending the magnet, if it is not too heavy, is to pass it through a paper stirrup (Fig. 69, *a*) which is suspended from a wood or brass stand (not an iron one) by a piece of unspun silk. The unspun silk has practically no directive force on the magnet, and moreover there is no friction, so that the magnet is quite free to turn in any direction in the horizontal plane. The arrangement is very susceptible to draughts, so that it is advisable to cover it with a glass bell jar while experiments are being made.

The end of the magnet which points approximately north is called the north pole; the other end is the south pole. Determine in this way the polarity of several bar magnets. Now suspend again one of the magnets, and approach the north pole of another magnet to the north pole of the suspended magnet. Record the results, and repeat the experiment with the south pole of the magnet. Note that like poles repel each other, and unlike poles attract.

Now bring up an unmagnetised bar of iron in turn to each end of the suspended magnet. Notice that in both cases the magnet is attracted. This affords a method of testing whether a piece of iron or steel is magnetised or not. The attraction is due to the fact that the nearer pole of the suspended magnet

induces a magnetic pole of opposite sign at the nearer end of the unmagnetised bar of iron, and the two poles then attract each other. The possibility of induced magnetism indicates a necessary precaution to be observed in testing the magnetism of a magnetic substance. If a bar of iron is only feebly magnetised, and we place it too near the pole of the suspended magnet, the poles induced in the specimen by the suspended magnet may be stronger than the actually existing magnetic poles in the specimen itself, so that the attraction may take place at both poles. To overcome this difficulty we must obviously bring the specimen up to the pole very gradually and from a considerable distance. If the end of the specimen

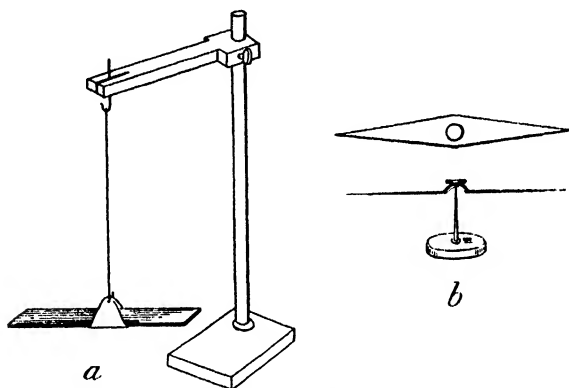


FIG. 69.—Methods of Suspending Magnets.

which we are bringing up has a weak pole of the same sign as the end of the suspended magnet which it is approaching, a small repulsion will be observed at some stage which may be followed by attraction as the distance is decreased. A non-magnetic substance, such as a bar of copper, ebonite, or wood, will not affect the suspended magnet in any way.

The method of suspension we have just been using, though very accurate, is not very convenient, as it is not very portable, and is extremely easily broken. The method of suspension generally adopted by instrument makers is to balance the magnetised "needle," as it is usually called, on a very sharp steel point. In expensive apparatus this point works in a jewel or piece of agate, but in cheaper forms it generally

works in a little brass cone. This introduces a certain amount of friction, and the needle may thus come to rest slightly out of its true direction. A little judicious tapping of the instrument will help to remedy this error, and should always be resorted to before taking a reading of the instrument. A needle suspended in this way is known as a compass needle (Fig. 69, *b*).

The straight line joining the two poles of a magnet is called the magnetic axis. The direction in which this line sets when the magnet is freely suspended is the magnetic meridian. A properly made compass needle is magnetised so that the magnetic axis passes through the two points of the needle. The magnetic axis, however, does not necessarily coincide with the axis of symmetry (or geometrical axis) of the magnet. In fact there will usually be a slight divergence between the two, which will have to be taken into account in very accurate experiments. We can, however, find the magnetic meridian, even when the direction of the magnetic axis is completely unknown.

EXPERIMENT 104.—To find the magnetic meridian, and the magnetic axis of a magnet.

Take a flat bar-magnet, and with soft wax fasten two small brass pins, one at each end of the magnet, as shown in Fig. 70, *a*. Suspend the magnet by a paper stirrup and silk fibre.

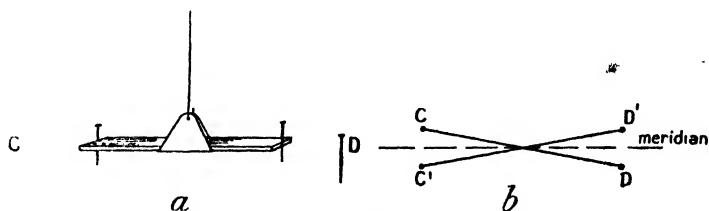


FIG. 70.—Determination of the Magnetic Axis of a Magnet.

Allow it to come to rest, and place two pins, C and D, near opposite ends of the magnet, so that the four pins are in exactly the same straight line. The magnet may be suspended just above a drawing-board covered with a sheet of white paper, and the pins may then be stuck directly into the board.

The magnet is then inverted in the stirrup, so that the face

which was uppermost is now underneath, and again allowed to come to rest. The magnetic axis of the magnet will again be pointing along the meridian, but unless the two pins on the magnet happen to be actually on the magnetic axis, the line joining them will not lie in the same direction as before, and the four pins will be no longer in line. Suppose that the line joining the pins made an angle θ with the magnetic axis, it will also make an angle θ with the magnetic meridian when the magnet is at rest. On turning the magnet over this line will still make an angle θ with the meridian, but on the opposite side of it. Place two more pins in the board to mark the line joining the two pins on the magnet when the latter has been inverted; remove the magnet and join up the pins by straight lines, as shown in the diagram (Fig. 70, *b*). The line bisecting the angle between these two lines is the magnetic meridian.

There are, of course, two such angles, one acute and one obtuse. The proper angle to bisect can be determined by remembering that the magnetic meridian must run approximately north and south. Since the magnetic axis of the magnet when freely suspended lies along the magnetic meridian, it can be determined by suspending the magnet again over the line PQ which marks the magnetic meridian, and marking on the magnet the points directly above this line.

EXPERIMENT 105.—To magnetise a steel knitting-needle.

FIRST METHOD (*by single touch*).—Place the steel-knitting needle on the bench. Take a bar-magnet and determine

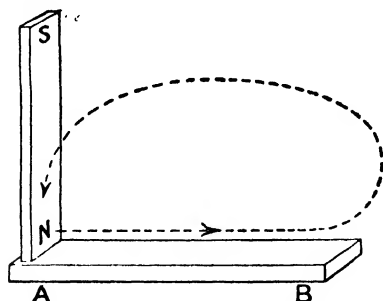


FIG. 71.—Magnetisation by Single Touch.

which is its north pole. Then draw the north pole of the magnet slowly and steadily along the needle from one end A (Fig. 71) to the other end B. Raise the north pole some distance above the needle, bring it down again on the end A, and repeat the rubbing. The rubbing must be done quite uniformly, as if the magnetic

pole is allowed to rest at any point on the needle a "consequent" pole is almost sure to be developed at that point.

After six or more strokes test the steel knitting-needle, and show that the end B where the magnet left the steel is a south pole. The rubbing can, of course, be done with the south pole of the bar-magnet, in which case the end B will be a north pole.

SECOND METHOD (*by divided touch*).—For this method two magnets of equal strength are required. Their polarities are tested, and the north pole of one is placed in contact with the south pole of the other on the centre of the needle to be magnetised (Fig. 72). They are then drawn apart until they reach their respective ends of the needle, raised up simultaneously, and replaced on the centre of the needle as before, the process being repeated several times. The end to which the north pole is taken becomes a south pole.

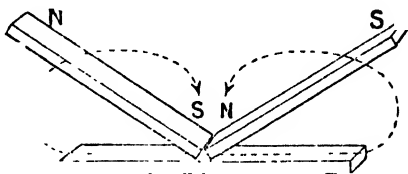


FIG. 72.—Magnetisation by Divided Touch.

THIRD METHOD (*by an electric current*).—The two previous methods are not very convenient, and skill is required to produce a uniform magnetisation. In practice, magnets are now made by means of an electric current. Take a piece of glass tubing about 30 cm. long and 2 cm. diameter and wind on it a coil of cotton-covered copper wire (No. 21 gauge is convenient), commencing at one end and winding as uniformly as possible until the other end is reached, and then back again to the original end, still winding in the same direction. Twist the two ends of the wires together to prevent the coil from unwinding, and tie down with tape. This forms what is known as a solenoid. To magnetise a piece of steel place it inside the glass tube, and connect the two ends of the wire to a storage cell, or a couple of bichromate cells, so that a considerable current (3 or 4 amperes) passes through the coil. Give the tube a shake and then immediately disconnect the battery. The current produces a strong magnetic field inside the tube, and the specimen becomes magnetised almost instantaneously, the process being assisted by the mechanical jar. This method produces a stronger and more uniform magnetisation than the two previous methods.

§ 31. THE MAGNETIC FIELD

ANY space in which magnetic force is exerted is called a magnetic field. The direction of the field at any point is the direction in which a single north pole would begin to move, while the strength of the magnetic field is the force which a unit north pole would experience if placed in the field. The field is thus measured in dynes per unit pole. As this is a unit of fundamental importance in magnetism, it has been given a special name, and is known as a *gauss*.

The direction of the field at any point can be determined by a small compass needle. The north and south poles are acted upon in opposite directions by the field, and the needle, therefore, turns round until it lies in the direction of the field.

The nature of a magnetic field is best exhibited by drawing lines of force. A line of force is a curve drawn so that its direction at any point is the direction of the magnetic field at that point. The lines of force due to a bar-magnet are a symmetrical set of curves running from the north to the south pole of the magnet. In practice the field due to the magnet itself is always superposed on the magnetic field due to the earth, so that the field as determined experimentally is the resultant of the two fields. It thus depends on the direction in which the magnet is placed relative to the magnetic meridian.

EXPERIMENT 106.—To plot the magnetic field in the neighbourhood of a bar-magnet.

Cover a drawing-board with a large sheet of white paper, and place a bar-magnet in the centre in some definite direction, say, in the magnetic meridian with its north pole pointing south. The direction of the magnetic meridian is determined by means of a compass needle (a long one, if possible) before the magnet is brought near the bench, and a line is ruled along the centre of the paper in the direction of the magnetic meridian. The

drawing-board is then clamped down to the bench, or weighted with lead blocks so that it is not liable to be moved during the experiment. Any movement of the board after the experiment has begun will, of course, completely spoil the result, and the experiment will have to be recommenced from the start. The magnet is then placed in the desired position, and secured by brass pins.

A small compass needle (the smaller the better) is placed on the paper near one of the poles, and when it has come to rest a dot is made with a sharp pencil immediately opposite each end of the compass needle. The compass is then moved bodily forward until the south pole is immediately over the dot which had been made opposite the north pole, and the position of the north pole is again marked by a dot. Continue the process up to the edge of the paper, and draw a smooth curve through all the dots so made. This curve will be a single line of force.

Now place the compass near the pole of the magnet again at a slightly different place, and trace another line of force in the same way. Continue until the nature of the field is quite clear from the diagram obtained. It will be found that there are certain points which are avoided by the lines of force. These are known as neutral points, and are points where the field due to the magnet is exactly equal and opposite to that of the earth. A compass placed exactly at a neutral point will be quite indefinite in its behaviour and may set in any direction. Extra lines of force should be plotted in the neighbourhood of the neutral points so as to fix their position as accurately as possible. The position of the neutral points depends on the direction the magnet makes with the meridian.

Plot in this way the fields due to a bar-magnet placed (a) in the magnetic meridian with its north pole pointing south, (b) in the magnetic meridian with its north pole pointing north, (c) at right angles to the magnetic meridian. Explain clearly why the fields are as you find them to be.

As an additional exercise clamp the bar-magnet in a vertical position with one end resting on the board, and plot the magnetic field in the plane of the board. Try and explain the features of *this* map.

§ 32. MAGNETIC UNITS

POLE-STRENGTH

IN order to make accurate measurements in magnetism we require to establish units in which to make and record our measurements. The system of units by which quantities are measured in magnetism, and also, as we shall see later, in current electricity, is known as the electromagnetic system of units and is founded on a definition of what we mean by a unit magnetic pole.

It can be shown experimentally that the force between two magnetic poles is inversely proportional to the square of the distance between them, and directly proportional to the product of their strengths. Thus if two poles are separated by a distance d in air, each exerts on the other a mechanical force which is directly proportional to the product of their strengths and inversely proportional to the square of the distance between them. If we take two equal poles and place them 1 cm. apart in air, each of the poles may be said to have unit pole-strength if the force which one exerts on the other is equal to our unit of force, that is, to one dyne. This defines a unit magnetic pole. If m and m' are the strengths of two poles measured in these units, then the force F which each exerts on the other is given by

$$F = \frac{mm'}{d^2} \text{ dynes}$$

where d is the distance between them.

The product of the strength of one of the two poles of a magnet into the distance between the two poles is called the magnetic moment of the magnet. The distance between the two poles of a bar-magnet is always less than the full length of the bar, and is generally not more than $\frac{5}{8}$ ths of this length. We can find the position of the poles of a magnet roughly by plotting its lines of force (see Experiment 108). As we shall

see later, it is possible to determine the magnetic moment of a magnet directly without knowing either the position or the strength of the poles. The magnetic moment of a magnet is also the couple required to hold it at right angles to a unit magnetic field. A unit magnetic field is said to exist at a given point if a unit magnetic pole placed at that point would be acted upon with a force of one dyne.

These definitions are not easy to put into practice with accuracy. In practice, we measure the strength of some uniform magnetic field (usually that of the earth itself) by the method indicated in Experiment 118, and use the known value of this field to determine the other quantities by a magnetometer method (see Experiment 109). The definition of pole-strength can be illustrated by the following experiment :

EXPERIMENT 107.—To determine the pole-strength of a magnet.

Two steel knitting-needles are selected of the same size and shape, and their weight is determined by a balance. They are then placed side by side in a solenoid, and magnetised by the third method in Experiment 105. They are then suspended by passing them through loops at the ends of two thin pieces of cotton of exactly the same length (about 80 cm.) which pass over a third knitting-needle supported by a firm wooden clamp. The threads must be adjusted so that the two knitting-needles lie exactly side by side in the same horizontal plane, with like poles pointing in the same direction, preferably in the magnetic meridian. When this adjustment has been secured, the threads may be kept from slipping by fastening them to the support with a touch of soft wax.

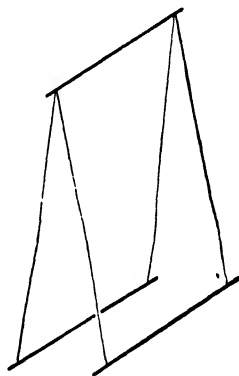


FIG. 73.—Measurement of Pole Strength.

Owing to the mutual repulsion of the like poles, the needles will repel each other, and will stand apart, as indicated in Fig. 73. Measure as accurately as you can with a boxwood scale the distance between the tips of the two needles. Both ends must be measured, and the mean distance taken. Let it be d . The

length l of the supporting thread from the centre of the needle to the centre of the support must also be measured. The pole-strength of the knitting-needles can now be calculated. Since the two needles have been magnetised simultaneously, we can assume that each has the same pole-strength m . The force between each pair of adjacent poles is then $\frac{m^2}{d^2}$, and since

there are two pairs of poles the total deflecting force is $2\frac{m^2}{d^2}$ acting horizontally. To balance this, there is the weight of the needle, equal to wg dynes (where w is the mass of the needle), acting vertically downwards through the centre of gravity of the needle. Taking moments about the support, we have

$$wg \cdot \frac{d}{2} = 2 \frac{m^2}{d^2} \cdot h$$

where h is the vertical height of the support above the plane of the needles, and is obviously equal to $\sqrt{l^2 - \left(\frac{d}{2}\right)^2}$. Thus, m can be calculated. d is usually so small compared with l that $\left(\frac{d}{2}\right)^2$ may be neglected in comparison with l^2 .

The following is a record of an experiment :

Mass of knitting-needle	= 4.50 gm.
d = distance apart after magnetisation	= 2.60 cm.
	2.62 „
	<hr/>
Mean	= 2.61 „

Length l of supporting thread = 40.2 cm.

$$h = \sqrt{(40.2)^2 - (1.3)^2} = 40.2 \text{ cm., to nearest mm.}$$

$$\therefore 4.50 \times 981 \times \frac{2.61}{2} = \frac{2m^2}{(2.61)^2} \cdot 40.2.$$

$$m^2 = \frac{4.50 \times 981 \times (2.61)^3}{4 \times 40.2} \quad m = 70 \text{ unit poles.}$$

EXPERIMENT 108.—To determine the pole-strength of a magnet from the position of the neutral point.

If the strength of the horizontal component of the earth's magnetic field in the laboratory is known, the pole-strength of

a magnet can easily be calculated from the magnetic maps plotted in Experiment 106. The simplest case is when the magnet has been in the meridian with its north pole pointing south. In this case the two neutral points will be directly north and south of the magnet and all the magnetic forces act in the same straight line.

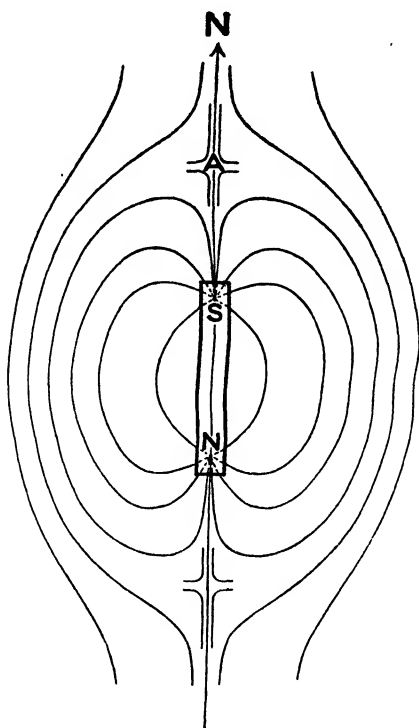


FIG. 74.—Determination of the Pole Strength of a Magnet from the Position of the Neutral Point.

Continue back the lines of force near each end of the magnet and find the points from which they appear to radiate. The majority of them will appear to radiate from two very small areas inside the magnet, and the centres of these areas may be taken as the positions of the corresponding poles. If the magnet has been correctly placed, these two poles and the two neutral points will all be in the same straight line (the

magnetic meridian). Let S and N (Fig. 74) be the two poles and A one of the neutral points. Measure the distances AS and AN.

The three forces acting on a unit north pole placed at A are : (1) The attraction due to the earth in the direction SA produced and equal to H, where H is the horizontal component of the earth's field ; (2) the attraction of the pole S, acting along AS and equal to $\frac{m}{AS^2}$; (3) the repulsion of the north pole at N acting in the direction NA and equal to $\frac{m}{AN^2}$, where m is the pole-strength of the magnet NS.

But since A is a neutral point the resultant of these three forces is zero. Hence

$$H = \frac{m}{AS^2} - \frac{m}{AN^2}$$

The value of m can be determined equally well from any of the other maps by any student who has a knowledge of mechanics. The principle of the method is exactly the same, but as the three forces are no longer all in the same direction they must be compounded instead of merely added or subtracted. The graphical method of the triangle of forces (see Experiment 32) is the most convenient.

§ 33. THE DEFLEXION MAGNETOMETER

A SMALL magnetised needle, when freely suspended, turns so that it lies along the resultant magnetic field, at the point where it is placed. Thus, when the earth's field is alone acting, the needle sets in the meridian. If now we apply another magnetic field, say by bringing up a bar-magnet, the needle will turn until it sets in the direction of the resultant field, that is to say, the field obtained by compounding the two separate fields, in accordance with the usual rules for compounding forces.

The simplest case is when the two fields are exactly at right angles to each other. Then if H is the horizontal component of the earth's field (the needle is always mounted so as to turn only in a horizontal plane, so that it is the horizontal component of the field which is alone effective) and F is the field due to the magnet, we have by the parallelogram law

$$\frac{F}{H} = \tan \theta \quad \text{or} \quad F = H \tan \theta$$

where θ is the angle between the needle and the magnetic meridian—that is to say, since the needle sets in the meridian before the deflecting field is introduced, θ is the deflexion produced by the field F .

Now the fields which we wish to measure generally vary rapidly with the distance, so that, if we wish to measure the field at some particular point, we must use a very short compass needle to ensure that the whole of it may be regarded as being at the point where the field is to be measured. The deflexions of a small needle cannot be read directly with any accuracy. The most accurate way is to fasten a small plane mirror to the compass needle, and measure the deflexions of a beam of light which is reflected from the mirror. The needle in this case is usually suspended by a fibre of unspun silk. This form of magnetometer is rather too delicate for an elementary laboratory. For elementary work the needle is carried on a sharp point, and has a long but light pointer attached to it at right angles to its length, the pointer moving over a scale

graduated in degrees, and usually enclosed in a flat brass box with a glass top (Fig. 75). The bottom of the box is often made of mirror glass, to enable parallax difficulties to be avoided. This constitutes the deflexion, or tangent, magnetometer.

For convenience, a pair of boxwood arms graduated in centimetres and millimetres are usually attached to the instrument,

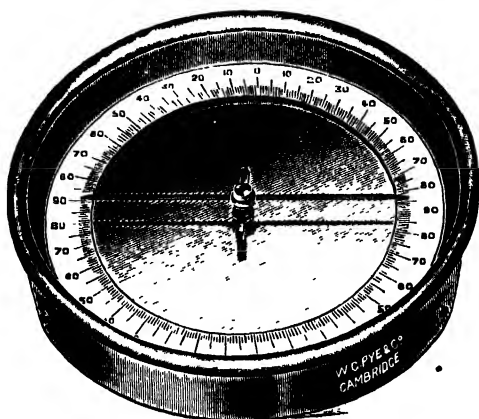


FIG. 75.—Deflexion Magnetometer.

their zeros coinciding with the point of support of the needle. This enables us to read off directly the distance from the centre of the needle to any object placed on the magnetometer arm.

RULES FOR SETTING UP THE MAGNETOMETER

In most experiments it is necessary that the magnetometer arms should be at right angles to the magnetic meridian. Remove all magnets and iron substances from the bench. Then turn the magnetometer round until the arms are parallel to the pointer. Since the latter is at right angles to the needle, which will necessarily be in the meridian, the arms will now be at right angles to the meridian. If the magnetometer is rigidly attached to the arms, it will be found that the two zeros on the circular scale are placed so that the line joining them is parallel to the arms, so that the two ends of the pointer will read zero when the adjustment is correct. If the magnetometer is separate from the arms, the scale should be turned into this position after the arms have been adjusted.

If the pointer is not quite straight, or if it is not attached exactly above the point of suspension, its two ends will not give exactly the same reading. Both ends of the pointer must always be read, and the mean of the two readings recorded. They should not differ by more than 1° . The accurate method of using the magnetometer is illustrated by the following experiment. The precautions described in this experiment should be taken, as far as they apply, in all magnetometer experiments.

EXPERIMENT 109.—To determine the magnetic moment of a bar-magnet.

Set up the magnetometer as described in the previous section. Measure the length of the bar-magnet, whose moment is to be determined, and find its middle point. Mark the middle point with a fine pencil mark. Place the magnet on the east arm of the magnetometer at such a distance from the needle that the deflexion produced is between 30° and 60° . Having obtained a suitable deflexion, it will be as well to slide the magnet along until its centre point coincides with some exact division on the scale. The length of the magnet must, of course, be parallel to the length of the scale. Tap the magnetometer gently, to make sure it is not sticking, and read both ends of the pointer.

Now, the centre of the bar may not be the centre of the magnet, that is to say, it may not be exactly half-way between the two poles. To allow for this, turn the magnet round so that the centre line is still on the same graduation, but with the opposite pole towards the magnetometer needle, and again read both ends of the pointer.

Finally, the zero of the magnetometer arms may not coincide exactly with the point of support of the needle. Take the magnet over to the other arm of the magnetometer, place its centre on the same graduation on this arm, and repeat the whole of the previous observations. You will thus have eight readings in all. These operations only eliminate small and unavoidable errors in the work. They do not correct errors due to careless or faulty settings.

The magnet in this experiment is in what is known as the "end-on position" to the magnetometer. It is shown in the text-books that for a magnet in this position the field F , at a distance d from the centre of the magnet, is given by

$$F = \frac{2Md}{(d^2 - l^2)^2}$$

where M is the magnetic moment of the magnet, and l half the distance between its two poles. Unless d is not very large in comparison with the length of the magnet, which will only be the case if the magnet is very feebly magnetised, l^2 will be very small compared with d^2 , so that a small error in l will not be of much importance. For instance, if d is five times l (that is to say, if the distance of the centre of the magnet from the needle is two and a half times the distance apart of the poles), l^2 will only be $\frac{1}{25}$ th of d^2 . Under such circumstances, if we know l to an accuracy of 10 per cent., our results will be correct to 1 per cent., which is as near as we can expect to get with this form of magnetometer. It will generally be sufficiently accurate to assume that the distance between the poles of the bar-magnet is $\frac{5}{8}$ ths of its whole length. l will thus be $\frac{5}{16}$ ths of the whole length of the magnet. Alternatively, the position of the poles can be determined by plotting the lines of force near the two ends of the magnet.

Now, with the deflexion magnetometer we have $\frac{F}{H} = \tan \theta$, where θ is the mean deflexion of the needle. Hence, substituting for F we have

$$\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan \theta$$

This determines M if H is given.

The results may be displayed as in the following record :

Length of magnet = 8.0 cm.

Distance of centre of magnet from needle = 15.0 cm.

Position of Magnet.		Deflexion θ .	
East arm, N pole facing needle .	.	42°·2	42°·4
" S " " " .	.	42°·6	42°·4
West arm, N pole facing needle .	.	42°·5	42°·3
" S " " " .	.	42°·7	42°·4

Mean deflexion, 42°·4.

$$\tan 42^\circ\cdot4 = 0\cdot913.$$

$$\therefore \frac{M}{H} = \frac{(15\cdot0^2 - (3\cdot3)^2)^2}{2 \times 15} \times 0\cdot913$$

$$= \frac{(225 - 11\cdot0)^2}{30} \times 0\cdot913 = 1392.$$

Taking H as 0.18 gauss (the correct value for your own laboratory should be used), $M = 253$ poles-cm.

If time allows, the experiment should be repeated, with the magnet at a different distance from the magnetometer needle.

EXPERIMENT 110.—To compare the magnetic moments of two bar-magnets.

If it is only required to compare the magnetic moments of two bar-magnets, the value of H will not be required. This is important, as the value of H in a modern laboratory is rarely the same as that in the open, owing to the iron used in the construction of modern buildings. Its value may, indeed, differ by as much as 10 per cent. in different parts of the same room, even when care is taken to work at a respectable distance from obvious masses of iron, such as hot-water pipes or iron standards. To compare the magnetic moments of two magnets, determine $\frac{M}{H}$ separately for each of them by the method of the previous experiment. H will, at any rate, be the same in both cases, if the magnetometer remains in the same position, and will cancel out. Thus, if

$$\frac{M_1}{H} = 1392, \text{ and } \frac{M_2}{H} = 1780, \text{ then } \frac{M_1}{M_2} = \frac{1392}{1780} = 1 : 1.26.$$

APPROXIMATE MAGNETOMETER FORMULÆ

If d is large compared with l , in practice if d is greater than ten times l or greater than five times the whole length of the magnet, we may, to the degree of accuracy with which we can work, neglect l^2 in comparison with d^2 , and the formula reduces to

$$\frac{M}{H} = \frac{1}{2} d^3 \tan \theta.$$

We can then arrange our experiments so as to simplify the calculation.

EXPERIMENT 111.—To compare the magnetic moments of two magnets by the equal-distance method.

The magnetometer is set up as before, and one of the magnets is placed at some suitable distance on one arm, so as

to produce a deflexion of about 30° . The deflexion is noted, and the experiment repeated in the other positions of the magnet, as in Experiment 109. The second magnet is then placed with its centre on the same graduation as that used for the first magnet, and the set of observations again taken. Since d is the same for both magnets and l is negligible, $\frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2}$. This method can be used even when the approximate formula would not be accurate, if the two magnets have the same length.

A more accurate method, when the length of the magnet is negligible, is as follows :

EXPERIMENT 112 — To compare the magnetic moments of two magnets by the null method.

Place one of the magnets on one arm of the magnetometer, at such a distance that a deflexion of about 30° is produced. Place the other magnet on the other arm of the magnetometer, with the same pole facing the needle, and slide it towards the needle until the deflexion is reduced to zero. Record the distances of the two magnets from the needle. Reverse both magnets, and repeat, keeping the centre of the first magnet on the same graduation. Interchange the two magnets, keeping the first magnet with its centre still on the same graduation, but on the other side of the scale, and repeat the previous observations. We shall thus have four values for d_2 , the distance at which the second magnet must be placed to produce the same effect on the needle as the first magnet, at a standard distance d_1 .

Since the two magnets neutralise each other, it is obvious that each in the absence of the other would produce the same deflexion. Hence $\tan \theta$ is the same for both magnets at these respective distances, and hence

$$\frac{M_1}{M_2} = \frac{d_1^3}{d_2^3}$$

d_1, d_2 being the means of the experimentally observed distances. (Strictly, each of the measured distances of the second magnet should be cubed, and the mean of the cubes determined. The distances will, however, differ so little that this more elaborate calculation is not necessary.)

EXPERIMENT 113.—Verification of the inverse square law by the deflexion magnetometer.

It must be noticed that whereas the field due to a single pole is inversely proportional to the *square* of the distance from the pole, the field due to a magnet is inversely proportional to the *cube* of the distance from the centre of the magnet, when the distance is large compared with the length of the magnet. This is a direct algebraical consequence of the law of inverse squares, so that if we verify the inverse cube law for a magnet, we also verify the inverse square law for magnetic poles. This verification may be performed by measuring the deflexion produced in the magnetometer needle by a small, well-magnetised bar-magnet, placed at different distances from the needle. The product $d^3 \tan \theta$ should be constant. The student who has mastered the principles of the previous experiment, should have no difficulty in working out this experiment for himself.

An excellent way of dealing with the series of observations is to plot a graph between the values of $\frac{1}{d^3}$ and the corresponding values of $\tan \theta$, or between d^3 and $\cot \theta$ ($= \frac{1}{\tan \theta}$). In either case the graph will be a straight line, showing that $\tan \theta \propto \frac{1}{d^3}$ or that $d^3 \tan \theta$ is constant.

Another method of verifying the inverse square law follows the method used by Gauss in his famous experiments on the subject. This consists in comparing the field produced at a given distance from a bar-magnet in the end-on position with that at the same distance from the same magnet in the broadside-on position. It is shown in the text-books that, assuming the law of inverse squares to be true for magnetic poles, the field at a distance d from a magnet in the broadside-on position is equal to $\frac{M}{d^3}$ when d is large compared with the half length of the magnet. Under the same circumstances the field in the end-on position is $\frac{2M}{d^3}$. That is, the field in the end-on position is twice as great as that in the broadside-on position.

EXPERIMENT 114.—Gauss's verification of the inverse square law.

Set up the magnetometer in the usual way. Determine the deflexion produced by a small well-magnetised bar-magnet at some suitable distance from the needle, using all four positions of the magnet in the usual way. A magnet about 6 cm. long should be employed, and should have been recently re-magnetised in a solenoid with a strong current. It should be placed with its centre at 25 cm. from the needle. The deflexion should be read to one-tenth of a degree, a pocket lens being used if necessary to magnify the scale.

The magnetometer is then turned so that the arms are exactly parallel to the magnetic meridian, that is, to the magnetised needle itself. If the circular scale is set so that the pointers read 0° when the arms are at right angles to the meridian, they will read 90° when the arms are in the meridian. If a long compass needle is available, it is more accurate to use it to mark the direction of the meridian on the bench (the magnetometer and all magnets are of course removed while this is being done), and to set the magnetometer arms parallel to this line.

The bar-magnet is placed with its centre immediately over the same scale division as before, *i.e.* the 25 cm. division, and with its length exactly at right angles to the scale. This adjustment is important, and should be made with a set square. The deflexion is read, the magnet reversed, and the reading again taken. These experiments are repeated on the other arm of the magnetometer. This broadside-on position of the magnet and magnetometer can of course be used for other experiments, but as the adjustment is more difficult and the deflexion smaller it has no advantages and the end-on position is generally preferred.

Record the results as follows :

Length of magnet = 6.0 cm.

Distance of centre of magnet from magnetometer needle = 25.0 cm.

Deflexion in the end-on position (four positions) :

$$\begin{array}{cccc} 28^\circ.5 & 28^\circ.7 & 28^\circ.4 & 28^\circ.5 \\ 28^\circ.4 & 28^\circ.6 & 28^\circ.2 & 28^\circ.3 \end{array}$$

$$\text{Mean } 28.4 = \theta_1.$$

Deflexion in the broadside-on position (four positions) :

$$\begin{array}{cc} 15^{\circ.2} \} & 15^{\circ.4} \} & 15^{\circ.1} \} & 15^{\circ.3} \} \\ 15^{\circ.0} \} & 15^{\circ.2} \} & 15^{\circ.0} \} & 15^{\circ.2} \} \end{array}$$

$$\text{Mean } 15^{\circ.2} = \theta_2.$$

$$\frac{\text{Field in end-on position}}{\text{Field in broadside-on position}} = \frac{\tan \theta_1}{\tan \theta_2} = \frac{\tan 28^{\circ.4}}{\tan 15^{\circ.2}} = \frac{0.5407}{0.2717} = 1.99.$$

Theoretical value = 2.

§ 34. THE VIBRATION OF A MAGNET IN A UNIFORM FIELD

WE have seen that if a suspended magnet is displaced from its equilibrium position it executes a series of oscillations before coming to rest. These vibrations, if the amplitude is not too great, can be shown to be isochronous, that is to say, the time of each complete vibration is exactly the same. The time of one complete vibration depends on the strength of the magnetic field, the magnetic moment of the suspended magnet, and on its dimensions and mass. It can be shown that if T is the time of vibration, H the field in which it is vibrating, and M the magnetic moment of the magnet,

$$T = 2\pi\sqrt{\frac{K}{MH}}$$

where K is a factor known as the moment of inertia of the magnet. For a straight bar-magnet, whose length is more than ten times its breadth, K is equal to $\frac{WL^2}{12}$, where W is the mass of the magnet, and L is its length from end to end. We can use this relation either to compare magnetic moments or to compare magnetic fields.

EXPERIMENT 115.—To compare the magnetic moments of two magnets by the vibration method.

As a simple example we may compare the magnetic moments produced in a steel knitting-needle by the various methods of magnetisation described in Experiment 105.

Take an unmagnetised knitting-needle and magnetise it by the method of single touch, making only a single stroke with the bar-magnet. Suspend the needle by passing it through a paper stirrup suspended by a piece of unspun silk, adjusting it in the stirrup until it hangs horizontally. Cover it with a bell jar, and give it a slight displacement by bringing up a

second magnet, and taking it away again. The oscillations should not be greater than 15° on each side of the equilibrium position. Take the time of fifty vibrations (by the method of Experiment 24), repeating the experiment at least three times. From these observations find the mean time of a single oscillation.

Now magnetise the needle as fully as possible by the method of single touch, taking care to stroke the needle in the same direction and with the same pole of the magnet as before. Suspend the needle, and determine its time of oscillation as before.

Now place the needle in the solenoid, and magnetise it by means of a strong electric current. (Experiment 105, Third Method.) It does not really matter which way the needle is placed in the solenoid if the current has the strength suggested, as it will wipe out any magnetism the needle may possess, and remagnetise it in its own direction. Again determine the time of oscillation. Record the results as follows :

Method of Magnetisation.	Time of Fifty Oscillations.	Mean time T of One Oscillation.	$\frac{1}{T^2}$
Single touch, 1 stroke .	181 sec. 182 sec. 181 sec.	3 62 sec.	0.076
Single touch, 20 strokes	124 sec. 124 sec. 123 sec.	2.48 sec.	0.163
Solenoid . . .	103 sec. 102 sec. 102 sec.	2.05 sec.	0.238

The numbers in the last column are directly proportional to the magnetic moment of the magnet, since K is obviously constant as the same needle is used throughout. Thus the magnetic moment produced by the solenoid is $\frac{0.238}{0.163}$ or 1.46 times that produced by twenty strokes in the single touch method.

The method can also be used to compare the magnetic moments of two different magnets if they are exactly of the same dimensions and mass. This is not usually the case, however, and the deflexion magnetometer method will generally be more convenient.

If the value of H is known, and the moment of inertia of the magnet is determined from its length and mass, we can, by substituting in the equation $T = 2\pi\sqrt{\frac{K}{MH}}$, actually calculate the magnetic moment of the magnet. In the case of the needle used above, $W = 4.55$ gm., $L = 16.0$ cm.

$$\therefore K = \frac{4.55 \times 16^2}{12} = 97.2.$$

Also $H = 0.18$ gauss.

$$\therefore \text{for magnetisation by solenoid, } 2.05 = 2\pi\sqrt{\frac{97.2}{M \times 0.18}}$$

$$M = 1650 \text{ poles-cm.}$$

THE VIBRATION MAGNETOMETER

For a given magnet the time of swing will be inversely proportional to the square root of the magnetic field in which it is oscillating. This can be expressed by saying that the strength of the field is inversely proportional to the square of the time of oscillation, or directly proportional to the square of the number of oscillations in a given time. A suspended magnet when used in this way to compare magnetic fields is known as a vibration magnetometer.

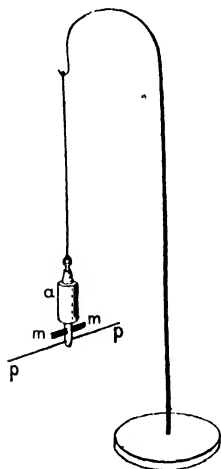


FIG. 76.—Vibration Magnetometer.

As the field to be measured may be a variable one, the magnet used must be a short one, so that we can regard both its poles as coinciding with the point at which the field is to be measured. If, however, the magnet is small, and therefore light, it may be attracted bodily towards a magnet whose field is being measured. To remedy this, and also to increase the time of swing, the small magnet mm is generally fastened in a horizontal position through the centre of a small brass cylinder a . The suspension is made by a piece of unspun silk fibre, so that the torsion of the suspension (which would affect the time of swing) shall be negligibly

small. In some cases a light aluminium pointer p is also fixed to the brass cylinder (generally parallel to the needle) so that the oscillations may be more easily perceived (Fig. 76).

EXPERIMENT 116.—To compare the horizontal magnetic field in different parts of the laboratory by the vibration magnetometer.

Set up the vibration magnetometer on a bench, and cover it with a glass bell jar to eliminate draughts. Set it in vibration by bringing up a small magnet and immediately removing it. The needle must oscillate about its point of suspension, *i.e.* in a horizontal plane, and not swing backwards and forwards as a whole like a pendulum. By means of a stop watch take the time of fifty complete oscillations. Repeat the observation to ensure greater accuracy. Make a sketch plan of the laboratory and mark on it the point where the observations were made. Now remove the magnetometer to another point in the laboratory and repeat the observations. If T_1 is the time of fifty vibrations in the first position, and T_2 the time of fifty vibrations in the second position, then

$$\frac{\text{Horizontal field in position 2}}{\text{Horizontal field in position 1}} = \frac{T_1^2}{T_2^2}$$

In this way the horizontal fields in different parts of the laboratory may be compared and their values recorded on the plan.

In most laboratories the presence of iron pipes, iron pillars, or girders will afford sufficient variety of values. If not, one or two large bar-magnets judiciously concealed immediately below the surface of one or two of the benches may be employed to add interest to the results.

If we wish to measure the field due, say, to a bar-magnet alone, the field due to the earth must be allowed for. The calculation will be simplest if the magnet is arranged so that its field is in exactly the same direction as that of the earth. The total field is then equal to the sum of the separate fields.

If H is the horizontal component of the earth's magnetic field and F the horizontal field due to the magnet alone, the total field when both are acting is $H + F$. If T_1 is the time of oscillation in the earth's field alone, and T_2 the time of oscillation in the combined field

$$\frac{H}{H + F} = \frac{T_2^2}{T_1^2} \quad \therefore F = H \left\{ \frac{T_1^2}{T_2^2} - 1 \right\}$$

EXPERIMENT 117.—To determine the magnetic moment of a magnet by the vibration magnetometer.

Cover a drawing-board with a sheet of white paper, and rule a line across the centre of the paper in the exact direction of the magnetic meridian. Mark a small cross near one end of the line, measure a distance of 15 cm. from the cross along the line, and make a second mark at this point.

Adjust the vibration magnetometer so that the centre of the vibrating needle is exactly over the first mark, and cover with the bell jar. Find accurately the time of fifty vibrations of the needle.

Now place the magnet whose moment is to be determined with its axis along the line marking the meridian, and with its centre on the 15 cm. mark. The magnet should be arranged so that the field it produces at the magnetometer is assisting the earth's field, that is to say, with its north pole pointing north if the magnet is placed south of the magnetometer. The centre of the magnet should be at the same horizontal level as the centre of the magnetometer needle.* If necessary the magnet may be placed on a small block of wood. Now again find the time T_2 of fifty complete oscillations. Then if F is the field due to the magnet

$$F = H \left\{ \frac{T_1^2}{T_2^2} - 1 \right\}$$

But the field F is that of the magnet in its end-on position.

Thus $F = \frac{2Md}{(d^2 - l^2)^2}$ where d is the distance of the centre of the magnet from the magnetometer (= 15 cm.) and l is half the distance between the poles of the magnet. Thus M can be calculated if H is known.

The results may be worked out as in the following experimental record :

Distance between magnetometer and centre of magnet = 15.0 cm.

Length of magnet = 8.0 cm.

Time of fifty vibrations in earth's field, 121.4, 121.8, 121.2 sec.

Mean = 121.4 sec.

Time of fifty vibrations in combined

field of earth and magnet, 90.6, 90.4, 90.8 sec.

Mean = 90.6 sec.

Field F due to magnet

$$\begin{aligned}
 &= H \left\{ \left(\frac{121.4}{90.6} \right)^2 - 1 \right\} = H \{ 1.798 - 1 \} \\
 &= 0.798H \text{ gauss} \\
 \therefore \frac{2 \times M \times 15}{(15^2 - 3.3^2)^2} &= 0.798H \\
 \frac{M}{H} &= 1220
 \end{aligned}$$

Taking H as 0.18 gauss, $M = 220$ pole-cm.

If a bar-magnet is suspended as in Experiment 115 and the time of oscillation T in the earth's field is determined as explained in that experiment, we have

$$T = 2\pi \sqrt{\frac{K}{MH}}$$

from which we get

$$MH = \frac{4\pi^2 K}{T^2}$$

If the *same* magnet is now placed on the arm of a deflexion magnetometer and an experiment carried out as described in Experiment 109, an accurate value for $\frac{M}{H}$ can be obtained. If

we divide the first of these values by the second we obtain a numerical value for H^2 from which H can be determined.

EXPERIMENT 118.—To determine the value of the horizontal component of the earth's magnetic field.

A cylindrical bar-magnet about 6 to 8 cm. in length and about 0.5 cm. diameter is convenient for this experiment. If it has not been recently magnetised it will be advisable to remagnetise it in a solenoid. It is suspended in a paper stirrup from a fibre of unspun silk, and placed beneath a bell jar. The time of one oscillation is determined as previously described.

The deflexion magnetometer is then set up, and using the same magnet used in the oscillation experiment the value of $\frac{M}{H}$ is determined. The exact formula is used for the calculation, the magnet being placed sufficiently near the magnetometer to produce a deflexion of at least 30°.

The moment of inertia of the bar-magnet is calculated by finding its mass W and length L . The length to be used in this calculation is the full length of the bar itself, not the distance between the poles. It should be measured accurately, with a pair of sliding calipers. If the bar is truly cylindrical and has the dimensions suggested, K will be equal to $\frac{WL^2}{12}$ to well within the limits of the error of the experiment.

The method of calculation is illustrated in the appended example :

Length of magnet = 8.0 cm.

Mass of magnet = 15.14 gm.

Time of 1 complete oscillation (from mean of three sets of 50)
= 8.21 sec.

$$\therefore \text{Moment of inertia } K = 15.14 \times \frac{(8.0)^2}{12} = 80.8 \text{ gm.-cm.}^2$$

$$\begin{aligned} MH &= \frac{4\pi^2 K}{T^2} \\ &= \frac{4\pi^2 \times 80.8}{(8.21)^2} = 47.4 \end{aligned}$$

Average deflexion of deflexion magnetometer

(mean of 8 observations) = $42^\circ.4$

Distance of centre of magnet from magnetometer = 15.0 cm.

$$\frac{M}{H} = \frac{(15.0^2 - 3.3^2)^2}{2 \times 15} \tan 42^\circ.4 = 1392$$

$$\therefore H^2 = MH \div \frac{M}{H} = \frac{47.4}{1392} = 0.0340$$

$$H = 0.184 \text{ gauss.}$$

§ 35. ELECTROSTATICS

EXPERIMENTS WITH A GOLD-LEAF ELECTROSCOPE

PRELIMINARY OBSERVATIONS

It is difficult to arrange elementary experiments in this subject which are capable of giving accurate numerical results. This is due partly to the difficulty experienced in the English climate of securing adequate insulation, as in our damp atmosphere it is by no means easy to prevent the charges which we place on our conductors from leaking away. A greater difficulty is that the exact measuring instruments required for electrostatic experiments (electrometers) are not only expensive and easily broken, but are, moreover, very difficult to use. For the most part we shall have to be satisfied with experiments which illustrate the phenomena of the subject, or which give us only approximate numerical results. This is, of course, no excuse for carelessness in performing them, or for not getting out of them as much as they are capable of giving. There is no part of the subject where greater care and skill is required.

THE GOLD-LEAF ELECTROSCOPE

The most convenient instrument for detecting and measuring electrical charges is some form of gold-leaf electroscope. A primitive type of electroscope which will nevertheless serve for a good many purposes consists of a pair of narrow metal leaves supported on one end of a metal rod, inside some vessel which serves to protect the leaves from draughts. The metal rod is insulated from the vessel to enable it to retain a charge, while the vessel itself should be conducting and connected to earth. An electroscope of this kind can be made in a very short time out of materials which are at hand in almost any laboratory.

EXPERIMENT 119.—To make a simple gold-leaf electro-scope.

Take a small conical flask with a cork to fit. Paste tinfoil on the base and sides of the flask, leaving a space in the middle through which the gold leaves can be observed. The flask is merely to serve as a container for the gold-leaf system, and any other form of container will do, if its surface is made conducting. Take a piece of stout brass or copper wire and bend it into a loop at one end as shown in Fig. 77. Bore a hole about 1 cm. in diameter through the cork, and find a

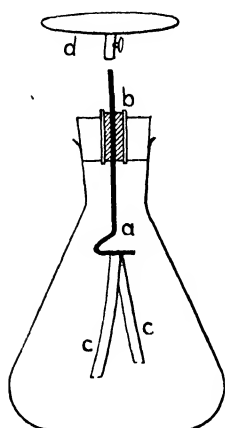


FIG. 77.—Simple Gold-leaf Electroscope.

piece of clean glass tubing to fit. The glass tube should project slightly through the cork, and its ends should be heated in a blow-pipe flame so that there is a slight contraction in the diameter of the tube at each end.

The glass tube is to hold the insulating substance by which the rod is to be insulated from its surroundings. Sulphur is by far the best insulator for general purposes, and has the advantage that it is easily cast into any required shape. It is, of course, not very strong mechanically, and where strength is required ebonite should be used. Glass is almost useless for insulating purposes in a damp climate. To cast our sulphur plug push the end of the wire through the middle of a piece of clean cardboard, and rest the end of the glass tube upon it so that the wire passes centrally through the tube. Take a stick of sulphur, scrape the outside to remove any dirt, and break a few small pieces into a clean evaporating dish. Now heat very gently until the sulphur melts to a clear mobile amber liquid. If the heating is continued until the sulphur becomes "treacly" its insulating properties will be to some extent impaired. Pour the liquid into the top of the glass tube. The cardboard at the bottom will prevent it from running out. When cool, the wire will be found to be firmly fixed in the centre of the glass tube by an insulating plug of sulphur. Any sulphur which may have flowed out of the tube can be cut away with a clean knife. It is quite soft for some hours after casting.

The plug is prevented from slipping out of the tube by the slight constriction which we have made at the ends.

For the leaves, a book of Dutch metal leaf, such as is used in cheaper kinds of gilding, will be required. Gold leaf is more sensitive, but it is much more difficult to handle than Dutch metal, and is not to be recommended to a beginner. The thinnest aluminium leaf can also be used. Take a sheet of the Dutch metal between the two pieces of chalked paper in which it is placed in the book, and with a pair of sharp clean scissors cut a strip about 2 mm. wide and the full length of the sheet, cutting through both the pieces of paper and the leaf. The top piece of paper is then carefully removed. Slightly moisten the middle of the horizontal bend in the wire with gum and press it down firmly on the exact centre of the strip. The leaf will adhere to the wire at this point, and on lifting the latter up the two halves will hang down side by side, and the whole can be inserted gently in the flask, as shown in Fig. 77.

For some experiments it is convenient to have a small brass plate on the projecting end of the electroscope. A circular plate can be cut out of sheet brass, and a binding screw (such as is used for connecting two wires together) soldered in the centre. When required the binding screw can be slipped on the projecting end of the wire carrying the gold leaf, and secured by tightening the screw.

The outside of the electroscope should be earth connected. This can be done efficiently by connecting the tinfoil to the water pipes by a copper wire. For most purposes, the bench on which the electroscope will stand is a sufficiently good conductor, and the case of the instrument may generally be regarded as earthed, unless special precautions are taken to insulate it.

If a charge is given to the insulated rod of the electroscope a difference of potential is established between the leaves and the case, and the leaves diverge. The divergence increases with the magnitude of the charge, but in the simple type of instrument described is not usually proportional to it, so that we cannot use the instrument for numerical measurements.

A considerable number of qualitative experiments can be made with the simple electroscope to illustrate the laws of electrostatics. Some of these are described below; others may be suggested by the statements in text-books on theoretical electricity.

EXPERIMENT 120.—To charge an electroscope, and to show that there are two kinds of electricity.

Rub an ebonite rod with flannel, and bring it near the cap of electroscope. The leaves diverge. Remove the rod without touching the cap. The leaves collapse again. Now bring up the rod once more and rub it gently on the cap of the instrument. (Take care that the rod is not sufficiently highly charged to break the leaves.) On removing the rod the leaves remain diverged, though not to the same degree as when the rod was in contact. Some of the electricity on the rod has been transferred to the electroscope by *conduction*. Charge the rod again by friction and bring it near the cap. The leaves diverge more widely. Thus when a charge of the same kind of electricity is brought near a charged electroscope the divergence of the leaves is increased.

Discharge the electroscope by touching the cap with the finger, and repeat the experiment, using a glass rod rubbed with silk. The glass rod may require to be warmed in front of a gas fire before it will hold a charge. A similar series of effects will be observed. Now charge the ebonite rod again by friction and bring it near the electroscope which has been charged by conduction from the glass rod. The leaves will be found to converge and may even, if the rod is well charged, collapse completely and diverge again. There is thus a difference between the electricity excited by silk on glass, and that excited by flannel on ebonite. The former was called vitreous, the latter resinous. They are now called respectively positive and negative for a reason which may be made clear by the following experiment :

EXPERIMENT 121.—On positive and negative electricity.

Place a small metal can (*e.g.* a copper calorimeter) on the cap of the electroscope, or, alternatively, insulate the can on a block of clean paraffin wax, and connect it to the cap by a copper wire. If a wire is used it must not touch anything except the cap and the can. Charge the glass rod and hold one end inside the can. The leaves will diverge. Now charge an ebonite rod and place one end inside the can together with the glass rod. As the charged ebonite approaches the divergence of the leaves grows less, and will probably at some point become zero. The electricity on the ebonite rod has

neutralised the effect of that on the glass rod, so that their combined effect is zero. This sort of thing is expressed in algebra by giving one quantity a positive sign (+) and the other a negative sign (-). It has been agreed to consider vitreous electricity as positive, and resinous electricity as negative.

Record all your observations fully, and state as completely as you can the deductions to be drawn from the phenomena.

These results provide a method of determining the sign of a given electrical charge.

EXPERIMENT 122.—To determine the sign of a given charge.

Charge the electroscope positively by conduction from a glass rod rubbed with silk. Now bring up another charged body, say a stick of sealing-wax rubbed with flannel. If the divergence of the leaves increases the body is positively charged. The body should be brought up slowly from some distance. Otherwise, if it is strongly charged with electricity of the opposite sign to that of the electroscope the leaves may collapse and open again so rapidly that their motion may not be noticed. If the leaves gradually collapse the body probably has a charge of the opposite sign to that on the electroscope. This is not necessarily so, as may easily be proved by placing the hand close to the cap of the electroscope. The leaves will converge, though the hand being an earth-connected conductor has no charge. Hence the only sure test of the sign of a charge is to obtain an increased divergence of the leaves. Thus, if on bringing up the charge the leaves converge, the experiment must be repeated with the electroscope charged with the opposite sign of electricity.

Test in this way the electrification produced when different solids, such as glass, sealing-wax, brass, sulphur, etc., are rubbed with silk, flannel, catskin, etc. In the case of conductors of electricity, such as brass or wood, the substance must be mounted on an ebonite handle to prevent the loss of charge to earth. The substance may then be flicked with the rubber, and the electrification tested. It may be found that the sign of the electricity developed on a given substance depends not only on the substance but also on the rubber employed. If the rubber is itself insulated by being tied round

the end of an ebonite rod, it will be found that the charge excited on the rubber is of opposite sign to that excited on the substance.

When the electricity developed is only feeble, it may be better to proceed as follows: Place the small metal can, as used in Experiment 121, on the electroscope, and discharge by touching the cap with the hand. Take, say, a piece of india-rubber and rub it briskly on a sheet of dry drawing-paper. Place the rubber without delay inside the can. The leaves will show a small divergence. Determine the sign of the charge by bringing up in turn (a) a positively charged rod, (b) a negatively charged rod. The electricity on the rubber is of the same sign as that of the rod which produces an increased divergence.

This method should be adopted in testing for the presence of small quantities of electricity, as the effect produced on the electroscope when the charge is placed inside the can is much greater than when it is merely held near the cap.

EXPERIMENT 123.—To charge an electroscope by induction.

A more convenient way of charging an electroscope is by the process known as induction. Bring a negatively charged ebonite rod near the cap of the electroscope. When the leaves are well diverged touch the cap with the finger. The leaves will collapse. Remove the finger and then withdraw the rod. As the rod is withdrawn the leaves will be seen to diverge, and will remain diverged. Test the sign of the electrification by bringing up in turn a negatively and a positively charged rod. The charge on the electroscope will be found to be positive. If a positively charged glass rod is used instead of a negatively charged ebonite rod the sign of the induced charge on the electroscope will be found to be negative.

The proof plane.—A useful little adjunct to the electroscope is known as the proof plane. It consists of a small metal disc attached to one end of an ebonite rod (Fig. 78), so that it may be moved from place to place without becoming discharged. The metal disc may be curved in various ways so as to fit more or less closely a variety of surfaces. To test the charge on a conducting surface the proof plane, which has been previously discharged, if necessary, is laid on the surface as

closely as possible, and is then transferred to the electroscope. While in contact with the charged body, the plane becomes effectively a part of its surface, and the whole of the charge on the portion of the surface covered by the plane is transferred to it.

If the proof plane is then placed inside a small metal can connected to the electroscope, or standing on its cap, and made to touch the interior, the whole of the charge on the plane will be transferred to the electroscope, and the proof plane, on withdrawal, will be found to be completely discharged. If the plane is placed on the cap of the electroscope part only of its charge will be given to the latter. The first method is, therefore, to be preferred. The charge conveyed to the electroscope is the charge on an area of the conductor equal to the area of the proof plane. Thus, if the same proof plane is employed it is a measure of the charge per unit area, that is to say, of the surface density of the electrification.

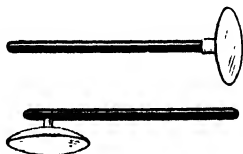


FIG. 78.—Proof Planes.

The reason why an electroscope can be used to measure charge is that the potential to which the gold-leaf system is raised when a charge is placed upon it is directly proportional to the charge. The deflexion of the leaves depends on the difference of potential between the leaves and the case. The latter, being earthed, is at zero potential. If the cap of the electroscope is connected to any conductor by a conducting wire, the gold leaves will be raised to the same *potential* as the conductor, and their divergence will measure, not the charge on the conductor, but its potential. The distinction between the two methods of using the electroscope is illustrated by the following experiment :

EXPERIMENT 124.—To investigate the induced charges and the potential on a conductor.

Take a conductor A, preferably of an elongated shape (Fig. 79), insulated by being mounted on an ebonite stem. Connect a flexible copper wire to the cap of the electroscope, and twist the other end of the wire round one end of an ebonite rod, to serve as an insulating handle. Bring near one end of the insulated conductor a second insulated conductor B, which is

strongly charged, either by means of an electrical machine or an electrophorus. Bring the wire from the electroscope into contact with any part of the first conductor. Note that the leaves diverge. Run the wire over different parts of the conductor. The divergence remains the same. Take a sample of the charge on the charged body by means of a proof-plane, and bring it near the electroscope while the wire is still in contact with the first conductor. Note that the divergence of the leaves is increased.

Disconnect the wire from the electroscope, and remove the latter a few feet from the charged body B. Recharge the latter from the machine. By means of a proof-plane, convey charges (*a*) from the end C of the first conductor nearer the charged body B, (*b*) from the opposite end D to the electroscope, and

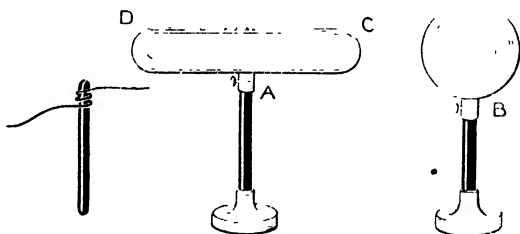


FIG. 79.—Investigation of the Laws of Electrostatic Induction.

determine their sign. The charge from the end near the charged conductor will have the opposite sign, that from the other end the same sign as the conductor.

Thus, while the potential of the conductor was the same at all points, the charges on its two ends were of opposite signs. Record the observations, and make what deductions you can from them.

Now connect the electroscope once more by a wire to the conductor A. Recharge the charged conductor B, and then earth the first conductor momentarily by touching it with a finger. Note that the leaves collapse completely. This will occur, no matter what point on the conductor is touched by the wire. Remove the wire, and again test the charge on the end of the conductor nearest the charged body. It will still be found to be charged. Determine the sign of this charge. Explain what light your experiments throw on the phenomenon of electrostatic induction.

§ 36. THE GOLD-LEAF ELECTROMETER

So far we have been using the electroscope merely as a qualitative instrument. The electroscope has been greatly improved in recent years, and some very important discoveries have been made by its means. An electroscope, with which useful quantitative experiments can be made, is shown in Fig. 80, and can be constructed without much trouble or expense.

It consists of a cubical metal box with windows at each end, through which the movements of the gold-leaf inside can be observed. These windows are covered with glass to keep out draughts. The brass rod (Fig. 80, *b*), which is to carry the gold-leaf, is beaten out over its lower half into a flat plate. It is insulated by a sulphur plug cast in a glass, or even a brass tube about 2 cm. in diameter, which fits closely in a second brass tube soldered into the top of the box. The system can thus easily be taken out when required to renew a broken leaf, or to replace a faulty insulation. A single leaf of Dutch metal is fastened by a touch of wax or gum to the rod at the place where the flat portion commences, and hangs vertically down when the electroscope is uncharged. When a charge is placed on the rod, the leaf is repelled. It is found that with this arrangement the deflexion of the leaf is very nearly proportional to the difference of potential between the leaf and the case, except for very small and very large deflexions.

To read the deflexion, the most accurate method is to use a long-focus microscope, with a graduated scale in the eye-piece. Another method is to paste a translucent paper scale on the front window, and to throw a shadow of the leaf upon it by

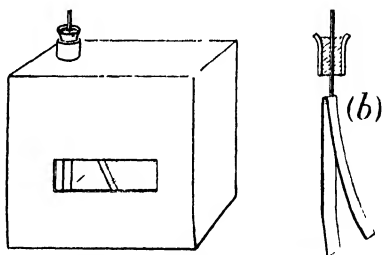


FIG. 80.—Gold-leaf Electrometer.

means of a pea-lamp placed behind the back window. A pocket flash-lamp answers the purpose quite well. A third method is to place a piece of plane-mirror glass so as to cover the lower half of the front window, and to place a horizontal paper scale in front of the mirror, and facing the electroscope. By adjusting the distance between the scale and the mirror, a virtual image of the scale will be seen which can be made to coincide without parallax with the position of the gold-leaf. If the scale is adjusted so that the graduations are seen along the top edge of the mirror, the position of the gold-leaf can be read off on the scale with fair accuracy.

This gold-leaf electrometer can be used for the qualitative experiments already described, and is used in the same way. It can also be used to give results of a more quantitative nature.

EXPERIMENT 125.—To investigate the distribution of electricity on a conductor.

This experiment can be performed qualitatively with the simple electroscope. The divergence of the leaves can be measured by using a plane mirror and a scale as described in the previous section. The gold-leaf electrometer will, of course, give more accurate results. In either case attach a small insulated metal can to the gold-leaf system, as already described, to receive the charges.

Insulate the conductor which is to be used for the experiment. If it is not mounted on an insulating support stand it on a clean block of paraffin wax. Conductors of various shapes can be used, *e.g.* a conducting sphere, a pear-shaped conductor, a hollow metal can. Charge the conductor by means of an electrophorus, or an electrical machine. Transfer the charge from a definite area of the conductor to the electroscope by means of a proof plane, and note the divergence. Discharge the electroscope and proof plane, and repeat with another portion of the surface. The conductor must be re-charged from time to time, as required. To make the experiment quite accurate the potential of the conductor should remain the same throughout the experiment. This can be done by attaching a second electroscope to the conductor by a wire, and adding charges to the conductor when required so that the divergence of the leaves of this second electroscope is kept always the same.

The simplest way of recording the results is to draw a

section of the conductor to scale. Make a mark on the drawing at each point which has been tested, and from the mark draw a normal to the surface making the length of this normal proportional to the corresponding deflexion produced in the electroscope. Join up the ends of all these normals and so obtain a graphic representation of the density of the charge on the conductor. The result for a hollow metal can is illustrated in Fig. 81. Note that in general the density is greatest where the curvature is greatest. Note also that there is no electrification inside a hollow conductor except just near the opening. Test the potential inside the can. It will be found to be the same as that of the outside.

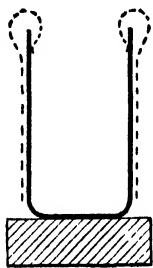


FIG. 81.—Distribution of Electricity on a Conductor.

The student will probably not have arrived at this stage without discovering that so-called insulators differ appreciably in their capacity for enabling a conductor to retain a charge. Even the sulphur insulation of the electroscope will not be quite perfect, and the gold-leaf system will gradually lose its charge as shown by the gradual collapse of the leaves, though this will probably not exceed $\frac{1}{2}$ mm. per minute, as long as the surface of the sulphur is free from dust. The resistance which different bodies offer to the passage of electricity through them can be compared in the following way :

EXPERIMENT 126.—To compare the resistances of different insulating substances.

Take lengths of, say, 20 cm. of silk and of cotton threads. To make the comparison a fair one as between different substances the threads should be of the same length and the same diameter, but the method will in any case enable us to compare the actual resistances of the bodies we are using. Connect one of the threads to the cap of the electroscope, and the other end to an earthed conductor. The case of the electroscope should also be earthed, and the end of the thread may be fastened to the same earthing wire. Charge the electroscope so that the leaf stands just beyond some definite division on the scale, and when the charge has leaked away until the leaf is just passing this division start a stop watch and take the

time taken by the leaf to fall through, say, ten divisions of the scale. Repeat this observation three times, and find the mean time taken.

Now disconnect the first thread and connect the second thread in its place. Repeat the experiment, commencing the timing always at the same division of the scale, and allowing the leaf to fall always through the same number of divisions. The resistances of the two threads are directly proportional to the times taken by the gold-leaf to cover the same ten divisions on the scale.

Investigate by the same method how the resistance of a thread of a given kind, say cotton, varies with the length of the thread. Care must be taken that the thread does not touch any other body except the cap of the electroscope at one end and the earth wire at the other.

MEASUREMENT OF CAPACITY

The capacity of an electrical conductor is the ratio of the charge to the potential. The gold-leaf electrometer will enable us to compare the capacities of different electrical systems. The gold-leaf electrometer will itself have a capacity. This may be determined as follows :

EXPERIMENT 127.—To measure the capacity of a gold-leaf electrometer.

A brass sphere of about 5 cms. diameter mounted on an ebonite handle will be required. It can be shown that the capacity of a conducting sphere is equal to its radius. Charge the electrometer and note the deflexion of the leaf, that is, the difference between the scale reading of one edge of the leaf and the scale reading of the same edge when the leaf is discharged. Discharge the sphere by touching it. Its potential is now zero. Touch the projecting rod of the electrometer with the brass sphere, holding the latter by its insulating handle. The electrometer will share its charge with the sphere, until the two are at the same potential. Measure this potential by again reading the deflexion of the leaf, which will be less than before.

Let C_1 be the capacity of the electrometer, and C_2 that of the sphere. C_2 is equal to the radius of the sphere, which can be measured by calipers, and is therefore known. Let V_1

be the initial potential, and V_2 the common potential after the contact. These are proportional to the deflexions d_1 and d_2 . Since no charge is lost in the process, the charge leaving the electrometer is equal to that given to the sphere. Hence

$$\begin{aligned} C_1 V_1 - C_1 V_2 &= \text{charge lost by electrometer} \\ &= \text{charge gained by sphere} \\ &= C_2 V_2 \end{aligned}$$

$$\begin{aligned} \therefore C_1 &= C_2 \frac{V_1}{V_1 - V_2} \\ &= C_2 \frac{d_1}{d_1 - d_2} \end{aligned}$$

The capacity of the electrometer being now known, that of another small conductor can be determined by the same method; using the small conductor in place of the brass sphere C_2 will now be the unknown capacity.

In a similar way the variation in capacity of a parallel plate condenser, with variation in the distance between the plates, can be investigated, the gold-leaf system being connected by a wire to the insulated plate.

§ 37. MEASUREMENT OF CURRENTS

ELECTRIC currents are generally measured by the magnetic effects which they produce. The unit of current on the absolute system is that current which, flowing in a conductor of unit length bent into the arc of a circle of unit radius, would produce a magnetic field of unit strength (1 gauss) at the centre of the circle. The practical unit of current, the *ampere*, is one-tenth of the absolute unit. It can be shown to follow from this definition that if we have a current of i absolute units flowing in a circular coil of n turns of wire of radius r cm. the magnetic field at the centre of the coils will be $\frac{2\pi ni}{r}$ gauss. This field will be at right angles to the plane of the coils. If the coils are placed vertical and parallel to the magnetic meridian, the field will be at right angles to that of the earth. Hence if H is the horizontal component of the earth's magnetic field a small compass-needle mounted at the centre of the coils will set at an angle θ to the magnetic meridian, such that

$$\frac{2\pi ni}{r} = H \tan \theta$$

This is the principle of the tangent galvanometer (Fig. 82).

The quantity $\frac{2\pi n}{r}$ which is a constant for a given coil is called the galvanometer constant, G . The current i is then given by

$$i = \frac{H}{G} \tan \theta \text{ in absolute units, or}$$

$$i = 10 \frac{H}{G} \tan \theta \text{ in amperes.}$$

The quantity $10 \frac{H}{G}$ by which the tangent of the deflexion θ must be multiplied to give the current in amperes is called the *reduction factor* of the galvanometer. Since it depends on H the reduction factor of the same instrument will differ in

different parts of the world, and *may* differ in different parts of the same laboratory.

If the coils are exposed so that their number and radius can be measured, the galvanometer constant can be determined by counting the number of turns and measuring their diameter with a pair of calipers. If the coils are concealed the maker usually supplies the value of the galvanometer constant.

In order to make the instrument adaptable to a wide range of purposes, it is frequently wound with several different coils



FIG. 82.—The Tangent Galvanometer.

connected to different binding posts on the base of the instrument. One coil may consist of a few turns of thick wire and small resistance. These are used for measuring large currents. Another coil, consisting of 40 or 50 turns of fairly low resistance will be useful for moderate or small currents. A third coil, having from 200 to 500 turns of fine wire and of high resistance, is useful when the instrument is required for comparing electromotive forces.

EXPERIMENT 128.—To plot the magnetic field due to the galvanometer coils.

Remove the magnetometer from the centre of the coils. (If the magnetometer is not removable, 10 or 20 turns

of cotton-covered copper wire, wound on a circular groove of about 20 cm. diameter, may be used for the experiment instead of the galvanometer coils.) By means of a compass-needle set the plane of the coils parallel to the magnetic meridian. Mount two small drawing-boards horizontally, one on each side of the coil, so that the plane of the boards passes through the horizontal diameter of the coil. Cover the boards with a sheet of drawing-paper, and pass a constant current round the coils by connecting them in series with a Daniell cell. The magnetic field can then be plotted by means of a small compass-needle, as described in Experiment 106.

The field will be the combined field of the coil and the earth, and will depend on the setting of the coils. The experiment may be repeated with the coil in other positions, for example, at right angles to the magnetic meridian. Note carefully any important differences between the fields of force in the two cases.

EXPERIMENT 129.—To set up and use a tangent galvanometer.

Replace the magnetometer in its place at the centre of the coils (if it has been removed), and set the plane of the coils parallel to the suspended magnetic needle of the magnetometer, as accurately as possible by viewing it from above the coils. The needle usually carries a long pointer at right angles to its length. This must not be mistaken for the needle itself, which is quite short. If the adjustment has been properly made, the coils will now be in the meridian. If the magnetometer scale is movable, rotate it until both ends of the pointer read zero, or, if this is impossible owing to defects in the construction of the instrument, until they both give the same reading, which should be recorded as the zero error of the instrument. Connect the terminals of the galvanometer to a reversing key, the other terminals of which are connected to the circuit in which it is required to measure the current, *e.g.* a Daniell cell and a resistance box. Allow the current to pass through the coils by closing the key, and measure the deflexion, reading both ends of the pointer and taking the mean. Now reverse the current through the galvanometer by means of the reversing key, and again read the deflexion. If this does not differ by more than 1° from the previous reading, the mean of the two

readings may be taken as the actual deflexion. If the difference is larger than this, the coils have not been accurately set in the meridian, and the instrument should be readjusted.

A reversing key should always be used with a tangent galvanometer, and readings made on both sides of the zero, as just described. Serious error may arise if this precaution is neglected.

If possible, the deflexion of the instrument should be between 30° and 60° . If the deflexion is small, it is difficult to read it accurately, and if it is large, the tangent varies so rapidly with slight change in angle, that a small error in reading the angle makes a large error in the calculated value of the current. If the experiment is one where we can choose what current we will use, it should be adjusted so that the deflexion comes between these limits. If the current is not under our control, we must select the set of coils which gives a deflexion as nearly as possible within these limits.

§ 38. EXPERIMENTS WITH A TANGENT GALVANOMETER

(THE method of using the instrument described in the previous experiment should be strictly adhered to in all measurements made with the tangent galvanometer.)

EXPERIMENT 130.—To determine the relation between the current sent through the tangent galvanometer by a constant cell and the resistance in the circuit.

Set up the tangent galvanometer with a reversing key, as described in the previous experiment, and connect a resistance

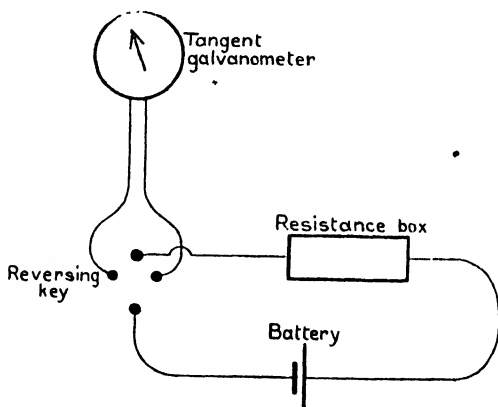


FIG. 83.—Determination of the Relation between Current and Resistance by the Tangent Galvanometer.

box in series with a Daniell cell (or an accumulator) to the other terminals of the key (Fig. 83). If an accumulator is employed, the resistance in the box should never be less than 1 ohm, unless the resistance of the galvanometer coils is known to be greater than 1 ohm. Otherwise so much current may flow that the cell may be injured. Note the deflexion, reading both ends of the pointer, and reversing the current. If the deflexion is greater than 70° , increase the resistance in the box until the deflexion is reduced to this amount. Then

record, in two parallel columns, the resistance in the box and the deflexions of the needle. Increase the resistance until the deflexion is about 60° , and again read the deflexion and the resistance in the box. Continue the observations until the deflexion has been reduced to about 20° .

By Ohm's law the current in the circuit is equal to the E.M.F. of the cell, which is constant, divided by the whole resistance of the circuit. This resistance is made up of the resistance of the galvanometer ($=G$), the resistance of the battery ($=B$, say), and the resistance in the resistance box ($=R$). Thus :

$$i = \frac{E}{G+B+R}$$

But $i = K \tan \theta$ where θ is the deflexion of the galvanometer and K is the reduction factor of the galvanometer. Hence

$$K \tan \theta = \frac{E}{G+B+R} : \text{or } G+B+R = \frac{E}{K \tan \theta} = \frac{E}{K} \cot \theta$$

Thus, since E and K are constants, if we plot a curve between the resistance R in the box and the corresponding value of the cotangent of the deflexion, the resulting graph should be a straight line. The values of the cotangents can be found in a book of mathematical tables. The point at which this line, when produced backwards, meets the axis of resistance will measure $B+G$, the sum of the resistances of the battery and galvanometer. If an accumulator has been used, B will be negligible, and the result will measure the resistance of the galvanometer.

If $B+G$ is small compared with the resistances used in the box the curve will pass very nearly through the origin. In this case the value of $B+G$ cannot be deduced with any accuracy. If possible a galvanometer with an appreciable resistance should be used for this experiment. The results may be recorded as follows :

Resistance in Box.	Deflexion θ (Mean).	Cot θ .
1 ohm	71°	0.344
5 "	63°	0.510
14 "	51°	0.810
26 "	40°	1.192
38 "	32°	1.600
61 "	23°	2.36

The graph between R and $\cot \theta$ was found on plotting on squared paper to be a straight line cutting the axis of R at a point corresponding to 10 ohms on the negative side of the origin (Fig. 84). $\cot \theta$ is, therefore, proportional to $R + 10$, that is, the resistance of the whole circuit is $R + 10$

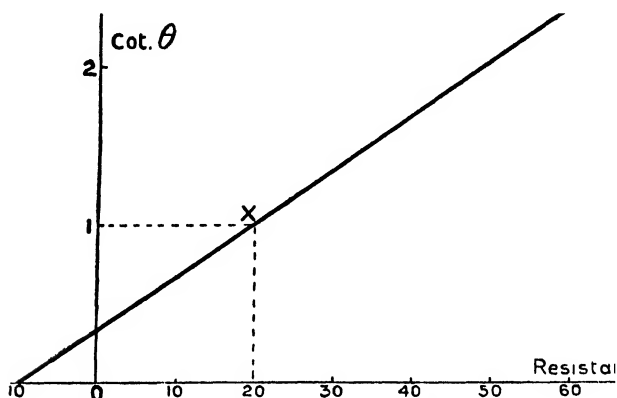


FIG. 84.—Graph showing the Relation between $\cot \theta$ and R , Experiment 130.

ohms, and the resistance of battery and galvanometer together (neglecting the resistance of the connecting wires, etc.) is equal to 10 ohms.

EXPERIMENT 131.—To determine the value of an unknown resistance by the tangent galvanometer.

The graph obtained in the previous experiment gives the relation between the cotangent of the deflexion and the "extra" resistance in the circuit (*i.e.* the resistance placed in the circuit in addition to the resistance of battery and galvanometer). Hence, if we remove the resistance box and substitute a coil of wire of unknown resistance, we can find the value of this resistance by measuring the deflexion θ produced in the galvanometer, and finding from tables the corresponding value of $\cot \theta$. Now find the point on the graph for which $\cot \theta$ has this value. The corresponding value of R will be the resistance of the unknown coil.

In the experiment graphed in Fig. 84 on substituting an unknown resistance coil marked X for the resistance box the

deflexion produced in the galvanometer was found to be 45° . The cotangent of 45° is 1.00. The ordinate of the graph in Fig. 84 is found to have this value when the abscissa is 20. The resistance of X is, therefore, 20 ohms.

Since the graph between R and $\cot \theta$ is known to be a straight line, it is only necessary for the purpose of measuring an unknown resistance to determine two points on the curve, one on each side of the resistance which it is desired to measure. Set up the galvanometer as described above, insert the unknown resistance in series with the battery, and measure the deflexion θ as accurately as possible. Now substitute the resistance box for the unknown resistance, and adjust the resistance until the deflexion θ_2 is slightly greater than θ_1 . Measure it accurately, and again adjust the resistance until a deflexion θ_3 , slightly less than θ_1 , is produced. The value of the unknown resistance can then be determined either by drawing a graph, as in the previous experiment, or by taking proportional parts as in the following example :

Resistance in Series.	Deflexion θ .	Mean.	Cot θ .
X	$\left\{ \begin{array}{cc} 48^\circ.2 & 48^\circ.2 \\ 48^\circ.6 & 48^\circ.6 \end{array} \right\}$	$48^\circ.4$	0.8878
48 ohms	$\left\{ \begin{array}{cc} 50^\circ.7 & 50^\circ.7 \\ 51^\circ.1 & 51^\circ.1 \end{array} \right\}$	$50^\circ.9$	0.8127
50 ohms	$\left\{ \begin{array}{cc} 45^\circ.4 & 45^\circ.4 \\ 45^\circ.8 & 45^\circ.8 \end{array} \right\}$	$45^\circ.6$	0.9793
$\therefore \frac{X - 48}{50 - 48} = \frac{\cot \theta_1 - \cot \theta_2}{\cot \theta_3 - \cot \theta_2} = \frac{0.8878 - 0.8127}{0.9793 - 0.8127}$ $= \frac{0.0751}{0.1666} = 0.45$ $\therefore X - 48 = 0.45 \times 2 = 0.90$ $X = 48.9 \text{ ohms.}$			

EXPERIMENT 132.—To compare the electromotive forces of two given cells by the tangent galvanometer. (First method.)

If E_1 is the electromotive force of one cell, B_1 the internal resistance of the cell itself, and R the resistance of the

galvanometer and the rest of the circuit, the current i_1 in the circuit is equal by Ohm's law to $\frac{E_1}{(R+B_1)}$, or since $i_1 = K \tan \theta_1$ where θ_1 is the corresponding deflexion

$$K \tan \theta_1 = \frac{E_1}{R+B_1}$$

Similarly for the second cell we have

$$K \tan \theta_2 = \frac{E_2}{R+B_2}$$

Thus

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{E_1}{E_2} \cdot \frac{R+B_2}{R+B_1}$$

Now B_1 will not usually be equal to B_2 and as both are unknown we cannot substitute directly in this equation to determine $\frac{E_1}{E_2}$. Both B_1 and B_2 will, however, probably be fairly small; the internal resistance of the cells commonly employed is not usually more than 2 ohms. Thus, *if the resistance R of the rest of the circuit is large compared with 2 ohms* we can, without any appreciable error neglect both B_1 and B_2 in comparison with R , and write

$$\frac{E_1}{E_2} = \frac{\tan \theta_1}{\tan \theta_2}$$

Set up the galvanometer using the high resistance coil of the instrument and place a resistance box in series with the galvanometer, so that the total resistance is at least 100 ohms, and more if possible. This will depend on the sensitiveness of the galvanometer, as the deflexions should not be less than 30° . Connect up the first cell and measure the deflexion. Replace by the second cell and again measure the deflexion. The ratio of the tangents of the two deflexions will be the ratio of the corresponding E.M.F.'s.

Resistance of galvanometer coils = 120 ohms.

Resistance in box = 100 ohms.

	Deflexion θ .	Mean.	Tan θ .
Deflexion using Daniell cell =	$\begin{Bmatrix} 35^\circ.4 & 35^\circ.6 \\ 35^\circ.4 & 35^\circ.6 \end{Bmatrix}$	$^\circ.5$	0.7107
„ „ Leclanché cell =	$\begin{Bmatrix} 46^\circ.3 & 46^\circ.5 \\ 46^\circ.3 & 46^\circ.5 \end{Bmatrix}$	$46^\circ.4$	1.0501
E.M.F. of Leclanché	$= \frac{1.0501}{0.7107} = 1.48$		
E.M.F. of Daniell			

The principle of this method is practically that adopted in the ordinary voltmeter. The voltmeter is really a sensitive ammeter in series with a high resistance, which is, of course, contained within the case of the instrument. As the resistance is large compared with that of a cell, the current through the instrument will be proportional to the E.M.F. of the cell. The instrument can thus have its scale graduated to read directly in volts. If, as is sometimes the case with the cheaper voltmeters, the resistance of the instrument is not large compared with that of the cell whose E.M.F. is to be measured, the reading of the instrument will depend, not only on the E.M.F. but also on the resistance of the cell, and may be considerably too low if the latter is large.

EXPERIMENT 133.—To compare the E.M.F.'s of two cells. (Sum and Difference method.)

Instead of making the resistance of the circuit so large that the battery resistance becomes negligible we may arrange the

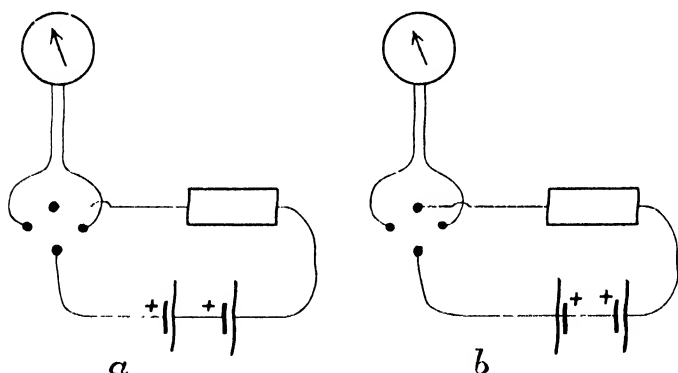


FIG. 85.—Determination of E.M.F. by Tangent Galvanometer. (Sum and Difference Method.)

experiment so that the resistance of the circuit remains constant throughout the experiment. This may be done by having both cells in series with the galvanometer throughout the experiment, but arranging that in one case they are acting in the same direction round the circuit, and in the other in the opposite direction.

Connect up the circuit as shown in Fig. 85 (a) so that the

positive pole of one cell is connected directly to the negative pole of the other. A resistance box may be inserted to adjust the current to a suitable value. If a resistance is used it must remain the same throughout the experiment. The deflexion is measured, and one of the cells is reversed, so that the positive poles of the two cells are now in contact (Fig. 85, *b*) and the deflexion again measured.

In the first case, the E.M.F. in circuit is the sum of the E.M.F.'s of the cells, in the second case it is the difference. Thus, if E_1 and E_2 are the E.M.F.'s of the two cells,

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{\tan \theta_1}{\tan \theta_2}; \quad E_1 = \frac{\tan \theta_1 + \tan \theta_2}{\tan \theta_1 - \tan \theta_2} E_2$$

since the resistance of the circuit is the same throughout the experiment.

Example :

	θ .	Mean.	Tan θ .
Leclanché and Daniell cell	$\{65^{\circ}.2 \quad 65^{\circ}.4\}$	$65^{\circ}.3$	2.1742
in series	$\{65^{\circ}.2 \quad 65^{\circ}.4\}$		
in opposition	$\{18^{\circ}.4 \quad 18^{\circ}.6\}$	$18^{\circ}.5$	0.3346
(Daniell cell reversed)	$\{18^{\circ}.4 \quad 18^{\circ}.6\}$		
$\therefore \frac{\text{E.M.F. of Leclanché}}{\text{E.M.F. of Daniell}} = \frac{2.1742 + 0.3346}{2.1742 - 0.3346} = \frac{2.5088}{1.8396}$			
	$= 1.37$		

The disadvantage of the method is that, unless the two E.M.F.'s to be compared are very different, the current when the cells are in opposition will be so much smaller than when they are acting in the same direction and the two deflexions will be of such different magnitudes that it is difficult to measure them both with accuracy on the same instrument.

§ 39. AMMETERS AND VOLTMETERS

PRACTICAL instruments for measuring current are known as ampere-meters, or ammeters. The most accurate of these work on the principle of a moving coil galvanometer. A coil of wire suspended with its plane parallel to the lines of force in a magnetic field tends to set itself at right angles to the lines of force if a current is passed round the coils. The turning-couple is resisted by a light spring, and the coil comes to rest in such a position that the turning-couple, due to the current, is equal to the restoring-couple produced in the spring. As the former increases with the current, the greater the current the greater will be the deflexion, and it is possible, by suitably designing the apparatus, to make the deflexion exactly proportional to the current. The magnetic field is supplied by a permanent horse-shoe magnet, and the deflexion is measured by a light pointer moving over a graduated scale. The latter is graduated, not in degrees but in amperes, or fractions of an ampere, so that the current may be read off directly from the scale.

The whole current to be measured is not usually sent through the suspended coil. The ends of the coil are usually connected by a small resistance, known as a shunt, which takes the bulk of the current, while only a small definite fraction passes through the coil. By changing the magnitude of the shunt, the fraction of the current passing through the coil can be altered, so that the same instrument can be used for measuring a very wide range of currents. Thus, if the sensitiveness of the instrument is such that a current of $\frac{1}{1000}$ th ampere ($= 1$ milliampere) flowing *through the coil* produces a deflexion of one division of the scale, and the coil is shunted with a resistance of $\frac{1}{10}$ th that of the coil itself, only $\frac{1}{10}$ th of the whole current will pass through the coils, and 1 milliampere through the coils will imply a current of 10 milliamperes through the rest of the circuit. That is to say, a reading of one scale division will indicate a

current of 10 milliamperes in the circuit. If the scale is graduated into 100 divisions, the instrument will now read currents up to 1 ampere. In a similar way, currents up to 10 amperes could be read on the same instrument by shunting the coil with $\frac{1}{9}$ th of its own resistance.

The instrument can be changed into a voltmeter by placing a suitable resistance in series with the coil. Thus, if resistance is placed in series with the coil so as to bring up its total resistance to 100 ohms, an E.M.F. of 1 volt connected to the terminals of the instrument will produce a current of $\frac{1}{100}$ th of an ampere, and a deflexion of ten divisions. Thus, ten divisions on the scale will correspond to an E.M.F. of 1 volt, and the instrument can be graduated to read as volts.

If a table galvanometer of the suspended coil type is available, these adjustments can be illustrated by the following experiments: (The suspended magnet type of table galvanometer is not usually suitable for these experiments, as its zero is very uncertain, and its scale far from uniform.)

EXPERIMENT 134.—To convert the given pointer galvanometer into a milliammeter.

Connect the galvanometer in series with a resistance box, a milliammeter, and a Daniell cell, and, before switching on the current, connect a piece of bare resistance wire (a piece of thin copper wire will probably serve if the galvanometer is fairly sensitive, and has a low resistance) as a shunt across the terminals of the galvanometer. Adjust the current by means of the resistance box until the milliammeter reads, say, 30 milliamperes. Alter the length of wire between the terminals of the galvanometer until the galvanometer reading is the same as that of the milliammeter. If the galvanometer reading is too low, more wire must be included between its terminals; if too large, the length of wire must be decreased. If a fairly long piece of bare wire is employed, the portion between the terminals can easily be adjusted by loosening one terminal, and drawing the wire gently through it. When the readings of the two instruments are the same, each division on the galvanometer will indicate a current of 1 milliampere. If the wire is then bent sharply upwards at the terminals, and the two ends soldered to copper strips, so that each angle in the wire comes exactly at the edge of the strip, the wire can be preserved as a permanent shunt for that particular galvanometer.

EXPERIMENT 135.—To convert the given pointer galvanometer into a voltmeter.

The galvanometer is placed in series with a resistance box and a constant cell, and the shunt made in the previous experiment is connected across its terminals. If an accurate voltmeter is available, the E.M.F. of the cell may be determined by connecting the poles to the voltmeter. If no voltmeter is available, the E.M.F. of the Daniell cell may be taken as 1.08 volts, or that of a properly charged accumulator cell as 2.10 volts. A large resistance is taken out of the box before the circuit is closed, to avoid risk of injury to the galvanometer. The resistance of the box is then decreased until the reading of the galvanometer is, say, ten times the known E.M.F. of the cell. For example, if an accumulator is used, the reading should be twenty-one scale divisions. Ten divisions on the galvanometer scale are then equal to 1 volt, and the instrument can be used as a voltmeter. A resistance coil of exactly the resistance in the box can be made, if desired, for use with the instrument. In a commercial voltmeter this resistance is, of course, permanently connected to the terminals, and enclosed in the case of the instrument. A little calculation will show that with the arrangement used in the experiment just described, the total resistance of the shunted galvanometer and the box must be 100 ohms. The student should prove this statement for himself.

Experiments 130 and 131 can also be performed (much more simply) by using an ammeter in place of the tangent galvanometer. In this case the current is read off directly on the ammeter, and a curve is plotted between the resistance in the box and the *reciprocal* of the current. This curve should be a straight line. The resistance of an unknown coil can also be measured in the same way as in Experiment 131.

The use of an ammeter for these experiments is theoretically less sound than the use of a tangent galvanometer, as we have to rely for the accuracy of our results on the graduations made for us by some unknown (and possibly inaccurate) instrument maker. In very accurate work ammeters are always calibrated before being used and their errors of reading determined.

CAUTION.—In the majority of ammeters the instrument is constructed so that the zero is at one end of the scale. The

current must therefore always be sent through the instrument in such a direction that the pointer moves along the scale. If the current is reversed the deflexion would be in the opposite direction, *i.e.* off the scale altogether, and the pointer would very probably be badly bent. The terminal marked + on the instrument is to be connected to the positive pole of the battery. The same remark applies to voltmeters.

§ 40. THE ELECTROCHEMICAL MEASUREMENT OF CURRENT

THEORETICALLY the unit of current is defined from its magnetic effects. Legally the unit of current is defined by the amount of silver which such a current would deposit from a solution of a silver salt in unit time. The international ampere is defined as the current which when passed through an aqueous solution of silver nitrate deposits 0.001118 gm. of silver in one second. For ordinary purposes the deposition of copper from a solution of copper sulphate in water can be used instead of the electrolysis of the more expensive silver salt, since a current which deposits 0.001118 gm. of silver per second will also deposit 0.000329 gm. of copper in the same time.

THE COPPER VOLTAMETER

A convenient and accurate form of copper voltameter is shown in Fig. 86. The two outer plates A, A, are connected together by copper wire and together form the anode, or positive electrode. The negative electrode or cathode is a similar plate of copper which is placed half-way between the two fixed plates, and parallel to them. This plate must be easily removable as it is the plate on which the deposition of copper is to take place. The liquid into which the plates dip is a solution of commercial crystallised copper sulphate in water of specific gravity 1.14, to which has been added one-tenth per cent., by volume, of concentrated sulphuric acid. The area of the cathode beneath the solution should be approximately 50 sq. cm. for each ampere of current. Attention to these points adds materially to the accuracy of the instrument.

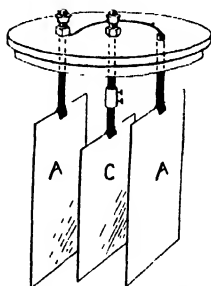


FIG. 86.—A Copper Voltameter.

With care an accuracy of about 1 part in 1000 can be obtained with a copper voltameter of this form, and it can thus be used to standardise a tangent galvanometer or ammeter. The use of the instrument is described in the following experiment :

EXPERIMENT 136.—To determine the reduction factor of a tangent galvanometer by means of a copper voltameter.

The tangent galvanometer is set up and adjusted, its terminals being connected to a reversing key in the usual way. A coil with a small number of turns should be used, as if

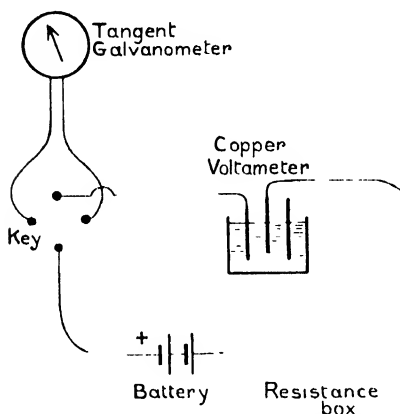


FIG. 87.—Determination of the Reduction Factor of a Galvanometer by a Copper Voltameter.

a large number of turns are employed the current will have to be very small to give a reasonable deflexion, and will consequently have to flow for an inconveniently long time to deposit a measurable weight of copper.

Two Daniell cells in series, or a single accumulator, are then connected through the copper voltameter and a resistance

box, to the other terminals of the reversing key (Fig. 87). Care must be taken that the plate which is to be weighed is connected to the negative pole of the battery. The proper connections are shown in Fig. 87.

The cathode plate of the voltameter is removed and cleaned by scrubbing it with silver sand and water until it is quite bright, and then rinsing well under the tap. The surface of the plate must on no account be touched with the fingers or a greasy mark may be made on which copper will not deposit. The plate is then replaced in the voltameter and the current is turned on and adjusted by means of the resistance box until the deflexion is between 40° and 50° . If the deflexion is too small even when there is no resistance in the box an

additional cell should be added to the battery. Allow the current to flow for five minutes to obtain a preliminary deposit of copper on the cathode. Then remove the cathode, wash it well under a gentle stream of water from the tap, and finally rinse it with distilled water. Press the plate firmly between two sheets of clean blotting-paper to remove as much moisture as possible, but do not rub it, as this might remove some deposit. Then complete the drying of the plate by holding it at some considerable distance above a lighted bunsen burner; the object being to dry the plate thoroughly without producing any oxidation of the deposited copper. The plate is then weighed as accurately as possible, and as the deposit which we are likely to get on our plate during the actual experiment will probably not weigh much more than $\frac{1}{5}$ th of a gram the weighing should be carried to three decimal places by means of a rider.

The cathode is now replaced in the voltameter, and at a given moment, which is carefully noted on an accurate watch or clock with a seconds hand, the current is switched on. The reading of the galvanometer is then taken, and is taken again at the end of each minute for which the experiment lasts. At the end of fifteen minutes the current through the galvanometer is reversed as quickly as possible by means of the reversing key, and the current is allowed to flow for a further fifteen minutes, the galvanometer reading being taken each minute as before. At the end of exactly thirty minutes from the moment when the current was started the circuit is broken and the current stopped.

The cathode plate is then removed from the voltameter, rinsed again first under the tap and then with distilled water, and dried and weighed, with the same precautions as before. The increase in weight gives the weight of copper deposited by the current in 30 minutes or 1800 seconds. If the galvanometer readings are not all identical the average deflexion θ of all the readings is taken.

If K is the reduction factor of the galvanometer the current i is equal to $K \tan \theta$. But the mass of copper deposited by a current i amperes in t seconds is 0.000329 it gm. Thus if m is the actual increase in weight of the cathode during the experiment

$$m = 0.000329 K \tan \theta \cdot t$$

$$K = \frac{m}{0.000329 \tan \theta \cdot t}$$

Record your results as in the following example :

Galvanometer used, No. 23 (4 turns).

Weight of cathode before experiment = 122.435 gm.

 " " after experiment = 122.617 "

Increase in weight = 0.182 "

Time of flow of current = 1800 seconds.

Average deflexion of galvanometer = 48°.6

(The actual deflexions at end of each minute should be recorded, but are omitted here to save space.)

$$\therefore \tan \theta = 1.1343.$$

Reduction factor of galvanometer

$$= \frac{0.182}{0.000329 \times 1.134 \times 1800} = 0.271 \text{ amperes.}$$

EXPERIMENT 137.— To determine the electrochemical equivalent of copper.

This experiment is identical with that just described. The only difference is in the calculation. In Experiment 136 the electrochemical equivalent of copper is assumed to be known and the results are employed to calculate the reduction factor of the galvanometer. If the latter is known, the results can obviously be used to calculate the electrochemical equivalent of copper. Calling this quantity e , we have from the previous equation

$$e = \frac{m}{it} \text{ or } e = \frac{m}{K \tan \theta \cdot t}$$

If an ammeter is used the current i will be read directly on the scale of the instrument and a reversing key will not be employed, an ordinary plug key being substituted. With a tangent galvanometer all the details will be precisely the same as in the previous experiment.

§ 41. MEASUREMENT OF RESISTANCE

THE resistance of a conductor is defined as the ratio of the potential difference between the ends of the conductor to the current flowing through it. Thus, if E is the applied P.D., and i is the current, the resistance R is given by

$$R = \frac{E}{i}$$

If E is measured in volts, and i in amperes, R will be given in ohms. By Ohm's law this ratio is constant for a given metallic conductor under given physical conditions. The resistance of a conductor can be determined directly from the definition if a suitable ammeter and voltmeter are available.

EXPERIMENT 138.—To determine the resistance of a conductor by means of a voltmeter and ammeter.

The resistance to be determined must have a value which will enable us to obtain suitable readings on both our instruments. Thus if the ammeter reads up to 1 ampere, the maximum current we can use will be 1 ampere. This will give a P.D. between the ends of the conductor of 1 volt if the conductor has a resistance of 1 ohm, of 2 volts if it has 2 ohms resistance, and so on. For accuracy both our instruments must give a measurable deflexion, so that unless we are furnished with a number of instruments covering different ranges we shall have to choose a resistance which will be suitable for the range of the instruments we are using.

Take a length of about 1 metre of resistance wire (manganin wire of about 2 ohms resistance per metre will probably be convenient), attach binding-screws to its ends, and hold it horizontally by placing the binding-screws in two wooden clamps (Fig. 88). Connect the wire in series with the ammeter, a resistance box, or, better still, a sliding rheostat, and one or two accumulators. Connect the voltmeter to the ends of the wire by means of the binding-screws, the positive terminal of the voltmeter being connected to the end of the wire which is

attached to the positive pole of the cell. The voltmeter must have a high resistance, as otherwise an appreciable fraction of the current which is measured by the ammeter will pass through the voltmeter and not through the wire. Complete the circuit, and read both voltmeter and ammeter. Reduce the current by taking out plugs in the resistance box or adjusting the rheostat, and again record the readings of the two instruments. Continue this process until the readings are too small to be taken accurately. A resistance box can usually

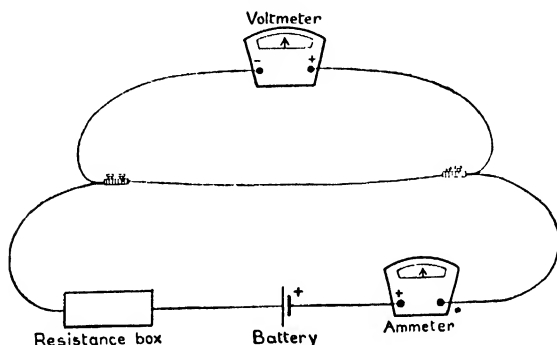


FIG. 88.—Determination of Resistance by Voltmeter and Ammeter.

only be adjusted in steps of 1 ohm. If this produces too big a change in the current, intermediate points can be obtained by inserting pieces of resistance wire of different lengths in the circuit. Their resistance will not require to be known.

The ratio of the voltage to the corresponding current measures the resistance of the wire. The ratio should be constant. Record the results as in the following example :

Current in Amperes.	P.D. in Volts.	$\frac{\text{Volts}}{\text{Amperes}}$
1'00	2'18	2'18
0'85	1'84	2'16
0'62	1'33	2'14
0'45	0'95	2'11
0'23	0'48	2'09
Mean . . . 2'14 ohms.		

This experiment is sometimes described as a verification of Ohm's law. This is not quite logical, as the use of a voltmeter assumes the accuracy of this law. If, instead of a voltmeter, we were to employ a quadrant electrometer or some other electrostatic form of potential measurer, the experiment would provide a verification of the law. Electrometers, when sufficiently sensitive to measure the potentials employed in current electricity, are too delicate and difficult to work to be used in an elementary course.

The measurement of a resistance by tangent galvanometer has already been described (Experiment 131). When great accuracy is required, however, some method based on the principle of the Wheatstone bridge is employed. The reason for this is that in the bridge methods our experimental conditions are varied until a sensitive galvanometer gives no deflexion when the galvanometer key is pressed down. Now it is much easier to decide whether a galvanometer needle does or does not move at all than to measure how far it has moved. A "null" method is therefore capable of greater accuracy than one which requires the actual measurement of a deflexion on a scale.

THE WHEATSTONE BRIDGE

If four resistances AB, EC, CD, DA are connected together to form a quadrilateral (Fig. 89), and a battery is connected to one pair of opposite corners, A and C, of the quadrilateral, and a galvanometer to the remaining pair B and D, it can be shown that the galvanometer will show no deflexion if the resistances are adjusted so that

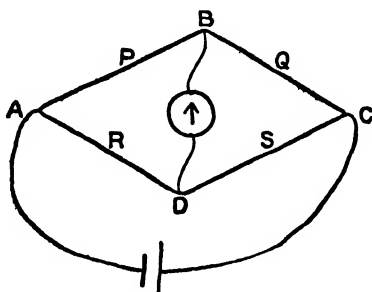


FIG. 89. —The Wheatstone Bridge.

$$\frac{\text{resistance of AB}}{\text{resistance of BC}} = \frac{\text{resistance of AD}}{\text{resistance of DC}}$$

When the bridge is balanced, *i.e.* when there is no current in the galvanometer, the resistance of any one of the conductors can be determined if that of the other three is known. In order to balance the bridge one at least of the resistances must be capable of being varied. The simplest method of applying the principle to the measurement of an unknown resistance is by the wire bridge or metre bridge.

The wire bridge consists of a resistance wire usually 1 metre long stretched over a metre scale. The ends of the wire are soldered to a thick copper strip ABC of negligible resistance in which there are two gaps fitted with binding-posts to which resistances P and Q can be attached as shown in Fig. 90. A battery consisting of a single Leclanché or Daniell cell is connected to binding-posts at A and C, while a sensitive

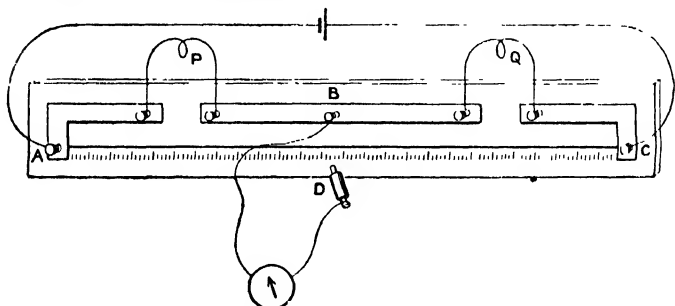


FIG. 90.—The Wire Bridge.

galvanometer is connected between a binding-post at B and a "jockey," which can be moved about so as to make contact at any point on the wire.

EXPERIMENT 139.—To determine the resistance of a given coil by the wire bridge.

The bridge is connected up as described, the unknown resistance Q being placed in one of the gaps in the bridge, and a known resistance P in the other gap. To preserve the galvanometer from damage during the preliminary setting it should be shunted by connecting a piece of resistance wire across its terminals. When the balance has been very nearly obtained, the shunt can be removed and use made of the full sensitiveness of the galvanometer in obtaining the final accurate balance-point. If this precaution is always taken when a sensi-

tive galvanometer is employed, it will prevent the risk of serious damage to the instrument.

The battery circuit having been closed, the jockey is caused to make contact in turn with different points on the wire. It will save time if this is done in a systematic manner. Make contact first near the end A, and note the direction in which the galvanometer deflects. Suppose it is to the right. Now make contact near C. If the deflexion is to the left the balance-point must lie between these two positions, and a deflexion to the right will mean that the point of contact needs to be moved towards the right. With this knowledge it is an easy matter to locate the actual balance-point. When a point has been found where the galvanometer shows practically no deflexion the shunt is removed, and the actual point determined as accurately as possible.

For accuracy this balance-point should lie somewhere near the middle of the wire. If it lies very near the end A of the bridge, P is very much smaller than the unknown Q, and a known resistance coil of higher resistance should be substituted for P, and conversely.

A balance having been obtained, P and Q are then interchanged and a new balance-point found. This will eliminate errors due to possible contact resistances in the bridge. Finally, the battery is reversed, so that the current flows in the opposite direction, and the two previous balance-points redetermined. This procedure eliminates errors due to the small thermo-electric forces which may exist at points in the circuit where two different metals are in contact. The errors introduced from this cause are generally very small, but the procedure will serve as a useful check on the accuracy of the previous working.

If P and Q are the resistances of the two coils, and R and S that of the portions of the wire between A and D and D and C, then

$$\frac{Q}{P} = \frac{S}{R}.$$

But if the wire is uniform, the resistances of the portions of the wire are proportional to their lengths. Thus

$$\frac{Q}{P} = \frac{d_2}{d_1} \\ \text{or } Q = P \cdot \frac{d_2}{d_1}.$$

The results may be worked out, as in the following record of an experiment :

Resistance in L. Hand Gap.	Resistance in R. Hand Gap.	d_1 cm.	d_2 cm	X ohms.
10 ohms	X	42.3	57.7	$\frac{57.7}{42.3} \times 10 = 13.6$
X	10 ohms	57.4	42.6	$\frac{57.4}{42.6} \times 10 = 13.4$
Battery Reversed				
X	10 „	57.4	42.6	$\frac{57.4}{42.6} \times 10 = 13.4$
10 ohms	X	42.4	57.6	$\frac{57.6}{42.4} \times 10 = 13.6$
Mean . . . = 13.5 ohms.				

The wire bridge method depends for its accuracy on the bridge wire being of uniform resistance throughout. The wire is exposed to risk of injury through a too vigorous use of the

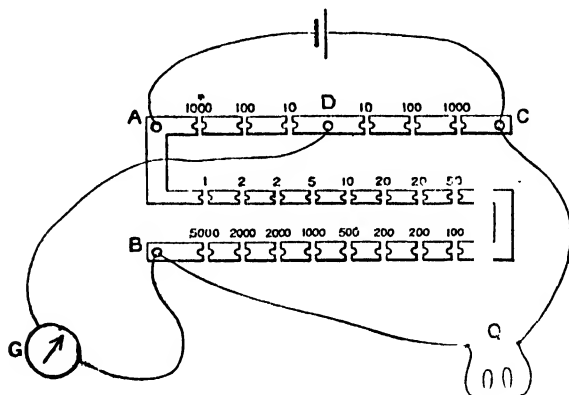


FIG. 91.—The Post Office Box.

jockey, so that even if uniform to begin with, it is likely to deteriorate in use. For this reason it has been largely replaced by a special box of resistance coils, which, from its having

been first designed for the measurement of the resistance of faulty telegraph wires, is generally known as the Post Office box. The arrangement of the box will be sufficiently clear from Fig. 91, which is lettered to correspond with Fig. 89.

The arms AD and DC are known as the *ratio arms*. With the resistances shown in the figure, the ratio $\frac{AD}{DC}$ or $\frac{R}{S}$ can be made to assume the values 1, 10, or 100; or, if required, $\frac{1}{10}$ th, and $\frac{1}{100}$ th. Q is again the unknown resistance, while the fourth arm of the bridge, AB, can be adjusted to give any whole number of ohms, from 1 to 10,000. When the bridge is balanced, we have

$$\frac{P}{Q} = \frac{R}{S},$$

from which Q can be determined, since the other three resistances can be read off on the box.

EXPERIMENT 140.—To determine an unknown resistance by the Post Office box.

Connect up the box as shown in Fig. 91, Q, the unknown resistance, being joined to B and C by thick copper leads, the resistance of which should be negligible compared with that of Q. Tapping keys, not shown in the figure, should be inserted in both the battery and the galvanometer circuit. The tapping key in the battery circuit is advisable, as the coils are apt to become slightly warm if the current is allowed to pass through them continuously, and their resistance may thus be slightly altered. A Leclanché cell should be used to furnish the current. The galvanometer should be shunted to commence with, as explained in the previous experiment.

Take the 10-ohm plug out of each ratio arm, making the ratio unity, and run over the rest of the plugs in the box to make sure that none of them are loose. A loose or badly fitting plug may add quite a considerable resistance to the box. Before any plug is taken out of the series part of the box, press down the battery key, and tap down the galvanometer key lightly, noting in which direction the galvanometer is deflected. Suppose it is to the right. A deflexion to the right, then, indicates that the resistance in this arm of the bridge is too low, and that more resistance must be inserted. Now take out the 5000-ohm plug, and note the deflexion when the keys are again worked in the same order. If the

deflexion is to the left, a balance will be found within the limits of the resistances available. The 5000-ohm plug is replaced, and the 100-ohm plug withdrawn. The deflexion of the galvanometer will indicate whether this resistance is too great or too small. In this way, the balance-point can rapidly be narrowed down until it is found to lie between two resistances differing by 1 ohm, the smallest step in the box. For example, we will suppose that it lies between 40 and 41 ohms, that is to say, with 40 ohms in the box, the deflexion is to the right; with 41 ohms, it is to the left. Since the ratio arms are equal, the resistance in the series part of the box AB must equal Q for a balance, and hence Q lies between 40 and 41 ohms.

Now make the ratio 10 : 1, by substituting 100 ohms for the 10 ohms in the arm AD. Then the resistance in AB must be ten times Q for a balance, and the balance-point will lie between 400 ohms and 410 ohms. By removing the proper plugs, we can ascertain that it lies between two new values differing by 1 ohm, say, between 403 and 404 ohms. The value of Q is thus between 40.3 and 40.4 ohms. By continuing the process with a ratio of 100 instead of 10, a second decimal place can be added.

It sometimes happens that on changing the ratio arms a balance is not obtained between the expected limits. This is nearly always due to dirty or badly fitting plugs, which may add an appreciable resistance to the series part of the box. Thus if in the example given, when a balance was obtained between 40 and 41 ohms, a badly fitting plug or plugs were introducing a contact resistance of, say, 2 ohms on their own account, the actual resistance would be between 42 and 43 ohms. With the 10 to 1 ratio, a resistance of between 420 and 430 ohms would be required for a balance, while with the 410 ohms out in the box, the total resistance of this arm would only be 412 ohms, even if the faulty plug were still in action. It will be seen that the effect of contact resistances becomes less formidable with the higher ratios. If the ratio arms are correct, the values obtained with the highest ratio will be the most accurate.

For practice, a number of different known resistance coils of different magnitudes should be used for the "unknown" resistance Q , until the box can be used with accuracy and certainty.

DETERMINATION OF SPECIFIC RESISTANCE

The resistance of a uniform wire is directly proportional to its length l , and inversely proportional to its area of cross-section α . We can thus write

$$R = \sigma \frac{l}{\alpha}$$

where σ is a constant which will depend on the material (and temperature) of the wire, and is known as the specific resistance of the material.

EXPERIMENT 11.—To determine the specific resistance of the material of a given wire.

To determine the specific resistance of a good conductor, such as copper, by the P.O. box is difficult. A copper wire of diameter $\frac{1}{2}$ mm. has a resistance of only 0.083 ohm per metre, so that something like 12 metres of the wire would be required to produce a resistance of 1 ohm. On the other hand, if we increase the resistance per metre by using thinner wire, the diameter cannot be determined with sufficient accuracy. The difficulties will be lightened if a wire of some more highly resisting alloy, such as manganin, is employed.

Take a sufficient length of the wire to have a resistance of at least an ohm and measure its resistance accurately by the P.O. box, or wire bridge. The wire itself can be attached to the binding-posts of the box, and no connecting wire will be required. Care should be taken, if the wire is an uncovered one, that different parts of the wire do not come in contact with each other, as a short circuit of this kind would materially affect the resistance. Having measured the resistance, the wire is bent up sharply where it leaves each binding-post. The length of the wire between those two bends will be length of the wire whose resistance has been measured. Measure this length by stretching the wire along a metre scale, taking care that there are no bends or kinks in the wire. The diameter of the wire is then measured by a micrometer screw gauge (Experiment 6), measurements being taken of two diameters at right angles to each other, and at several points along the wire. * As the main portion of the error is likely to occur in this measurement, the greatest care should be taken to obtain an accurate reading.

The result may be worked out as follows :

Resistance of manganin wire by P.O. box = 1.27 ohms.

Length of wire between kinks = 95.4 cm.

Diameter of wire = 0.661 mm. 0.663 mm. 0.665 mm.

0.660 „ 0.661 „ 0.663 „

Mean = 0.662 mm. = 0.0662 cm.

Area of cross section = $\frac{\pi}{4}d^2 = \frac{\pi}{4}(0.0662)^2 = 0.00344$ sq. cm.

∴ Specific resistance of $\frac{R \propto}{\text{manganin} \quad l}$

$$= \frac{1.27 \times 0.00344}{95.4}$$

$$= 4.58 \times 10^{-5} \text{ ohms per cm. cube.}$$

§ 42. THE POTENTIOMETER

JUST as resistances are measured more accurately by a null method such as the Wheatstone bridge than by a method in which a deflexion has to be measured, so electromotive forces can be compared more accurately by a null method such as is supplied by the potentiometer than by a voltmeter or tangent galvanometer. The potentiometer is based on the fact that if a constant current is passed through a long uniform wire the fall of potential along the wire will be uniform, so that the difference of potential between two points on the wire is proportional to the length of wire between them. The electromotive forces to be compared are balanced against the potential difference between two points on the wire. Thus if d_1 is the length of potentiometer wire required to balance the P.D. between the poles of a cell of E.M.F. = E_1 , and d_2 the length of wire required to balance that between the poles of a cell of E.M.F. = E_2 , then

$$\frac{E_1}{E_2} = \frac{d_1}{d_2}$$

The simplest form of potentiometer consists of a long uniform wire of fairly high resistance (preferably of manganin)

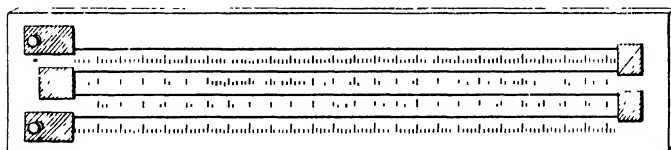


FIG. 92.—Single Wire Potentiometer.

stretched over a divided scale. To obtain great accuracy the wire is usually at least four metres long, and is then, for convenience divided into four equal lengths, each one metre long, connected at the ends as shown in Fig. 92, so as to form one continuous wire. The wire is soldered to binding-posts at

each end, and a jockey is arranged so that contact can be made at any point on any of the four wires. A potentiometer of this kind can very easily be made in the laboratory, and the scale can be made of squared paper (divided into mm. squares) which can be pasted below the wires, and numbered by hand.

EXPERIMENT 142.—To compare the E.M.F.'s of two cells by the potentiometer.

The battery which is to supply the current for the potentiometer is connected directly to the two ends of the potentiometer wire. This battery *must* have a higher voltage than that of any of the cells to be compared. In most cases a single accumulator will suffice, as the majority of primary cells in common use have a smaller E.M.F. than an accumulator. If

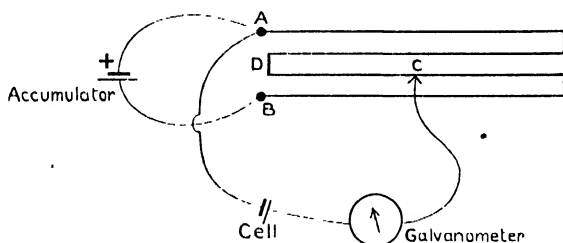


FIG. 93.—Comparison of E.M.F.'s by Potentiometer.

one of the cells to be compared is itself an accumulator, or a bichromate cell, it will be necessary to use two accumulators in series to provide the current.

One of the cells to be compared has its positive pole connected directly to the same binding-post on the potentiometer as the positive pole of the battery supplying the current. Unless the two like poles are connected together it will be impossible to obtain a balance. The negative pole of the cell is connected through a tapping key and a galvanometer to the jockey (Fig. 93). A high resistance (say 1000 ohms) should be included in series with this circuit in order to protect the galvanometer and battery from damage while the approximate position of the balance-point is being found. When this point has been reached the resistance is taken out of the circuit to increase the sensitiveness for the final balance. The galvanometer might be protected by shunting it as in

Experiment 139. The high resistance is, however, to be preferred for potentiometer work as it ensures that the cell which is being tested shall not give an appreciable current at any stage in the test, and thus prevents all fear of the cell polarising.

The jockey is touched down near one end of the wire, and the direction of the deflexion is noted. The jockey is then transferred to the other end of the wire, when the deflexion should be in the opposite direction. If this is not the case either the connections are wrong, or the battery supplying the current is not sufficiently strong. Assuming that all is correct, the jockey is placed at different points on the wire until an approximate balance is obtained. The high resistance is then taken out (or the galvanometer shunt is removed) and an accurate balance-point obtained. The length of potentiometer wire between the jockey and the end A of the wire to which the other terminal of the cell is attached is measured.

Care must be taken not to overlook a whole length of wire when reading the distance. If, for example, a balance is found with the jockey at C (Fig. 91), the length to be measured is AC, that is, two whole lengths of wire (2 metres), plus the length DC.

The second cell is then put in place of the first, and the operations are repeated. As the process takes time, and as the accuracy of the result depends on the potentiometer current remaining constant, the first cell is then substituted for the second, and the balance-point is verified. Should it have altered appreciably the second cell is again measured, the alternations being repeated until a constant result is obtained. If the two readings for the first cell only differ slightly from each other, the mean may be taken as correct. Record the results as in the following examples :

Cell Used.	Potentiometer Reading
Daniell	198.2 cm.
Leclanché	272.4 „
Daniell	198.0 „
$\frac{\text{E.M.F. of Leclanché}}{\text{E.M.F. of Daniell}} = \frac{272.4}{198.1} = 1.37.$	

If we assume that the E.M.F. of the Daniell is 1.08 volts, that of the Leclanché is $1.08 \times 1.37 = 1.48$ volts. The Daniell cell is generally taken as the standard cell for elementary work. Its E.M.F. is, however, not quite invariable, as it depends on the strength of the sulphuric acid employed in the inner porous pot. A solution of 1 part of concentrated acid to 12 parts of water gives an E.M.F. of 1.08 volts if the copper sulphate solution is saturated.

EXPERIMENT 143.—To compare the P.D. between the poles of a Daniell cell when it is on open circuit, and when it is short-circuited by a known resistance, and to deduce the internal resistance of the cell.

Connect up the potentiometer in the usual way, and find the balance-point for the Daniell cell as in the previous experiment. Now without altering the connections of the cell,

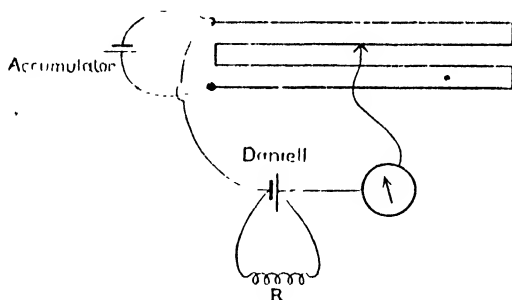


FIG. 94.—Measurement of Internal Resistance of a Battery by Potentiometer.

connect a resistance coil of 1 ohm to its terminals so that the cell sends a current through the coil, and again determine the balance-point. The connections are shown in Fig. 94. The new P.D. will be appreciably smaller than the former. Let d_1 be the first length of wire and d_2 the second, E_1 the P.D. between the poles of the Daniell when it is not giving a current and E_2 the P.D. when the poles are joined through the resistance. If R is this resistance and B the resistance of the Daniell cell, then the current i through the resistance R will be equal to $\frac{E_1}{(R+B)}$. The difference of potential be-

tween the ends of the resistance R will therefore be Ri , or $\frac{E_1 R}{R+B}$. But this is obviously the potential difference which we are measuring with the potentiometer when the cell is short-circuited by the resistance. Thus

$$E_2 = \frac{E_1 R}{R+B}$$

$$B = R \left(\frac{E_1}{E_2} - 1 \right)$$

$$= R \left(\frac{d_1}{d_2} - 1 \right)$$

Example :

	Potentiometer Reading.
Daniell cell on open circuit	196.4
Shorted through 1-ohm coil	108.6
On open circuit	197.0
Mean on open circuit	= 196.7

$$B = 1 \left(\frac{196.7}{108.6} - 1 \right) = 0.81 \text{ ohm.}$$

The experiment may be repeated using coils of 2 and 3 ohms resistance instead of the 1-ohm coil. The resistance of the Daniell cell generally increases as the current which it is giving decreases.

The experiment may also be performed, less satisfactorily, using the method of Experiment 132 for determining the potentials. It is obvious, however, that if this method is employed the resistance of the galvanometer used must be high.

EXPERIMENT 144.—To study the behaviour of a Leclanché cell.

The E.M.F. of the Leclanché is compared with that of a Daniell as in Experiment 142. The Leclanché is then short-circuited by a piece of copper wire for 1 minute and its E.M.F. again compared with that of the Daniell cell. The E.M.F. of the Leclanché will be found to have fallen. The Leclanché cell polarises when allowed to give a large current. Short-circuit the cell for a further four minutes, and again

determine the E.M.F. The progressive polarisation of the cell can thus be studied.

Now allow the cell to rest for two minutes and determine its E.M.F. Continue this process, testing the E.M.F. approximately every two minutes, and plot a curve to show how the recovery of the cell varies with the time since the short-circuiting ended.

ADDITIONAL EXERCISES AND EXAMINATION QUESTIONS

1. Magnetise a steel needle so that it has more than two poles. Place it in the magnetic meridian and plot the resultant field of force.

2. Magnetise the two pieces of knitting-needle, one of which is twice the length of the other, at the same time in the same solenoid. Compare their magnetic moments.

3. Determine the pole-strength of the given magnet. The value of the horizontal component of the Earth's magnetic field is given. (The most accurate method is to determine the magnetic moment of the magnet by a deflexion magnetometer, and then to determine the position of the poles by compass-needle.)

4. Place the given bar-magnet in the magnetic meridian, with its north pole pointing (α) South, (β) North. Plot sufficient of the field in each case to determine the position of a neutral point. Measure the distance of the neutral points from the centre of the magnet.

5. Plot the horizontal magnetic field in the neighbourhood of a vertical wire carrying an electric current. Discuss any features of interest in your map. (This experiment requires a fairly large current, say 2 or 3 amperes. The vertical wire should be at least 50 cm. long, and leads conveying the current should be kept well away from the drawing-board, as they will also produce magnetic fields. The plane of the drawing-board should pass approximately through the middle of the vertical wire.)

6. Arrange a horizontal wire about 1 metre long in the magnetic meridian and pass a constant current of about 2 amperes through it. Place a magnetometer beneath the wire

and determine how the deflexion of the magnetometer needle varies with the distance of the wire from the needle. Plot a graph between the distance and the cotangent of the angle of deflexion.

(The wire may be held in two stands, and the current leads to and from the wire must be kept well away from the magnetometer. The distances may be measured from the glass top of the magnetometer case and the distance of the needle from the glass top measured subsequently, using the reflection of a vertical scale in the glass top. The graph should be a straight line. The experiment is not difficult if a sufficiently large constant current can be provided, say by an accumulator cell.)

7. Arrange an experiment to show that when two bodies are electrified by rubbing them together the charges developed are equal and opposite.

8. Connect in series a resistance box, a tangent galvanometer of known resistance, and a storage cell of negligible resistance. Assuming Ohm's law to be true, make experiments to find how far the tangent law is obeyed by the given galvanometer. Represent your results by a graph.

9. Compare the resistance of the two given equal coils when in parallel with their resistance when in series.

10. Determine what length of the given wire would be required to make a 10 ohm coil.

11. Determine how the resistance of the given coil of copper wire varies with the temperature. (A coil of thin cotton or silk covered wire may be wound on a piece of glass tube, and should have a resistance at room temperature of not less than 2 ohms. It may be heated by being placed in a bath of machine oil, the temperatures being taken by a thermometer.)

12. By means of a voltmeter and ammeter find how the current through the given electric filament lamp varies with the potential difference between the terminals of the lamp.

(A 4-volt "pea" lamp such as is used in a pocket flash-lamp is convenient. Current may be supplied from two accumulators in series and varied by a resistance box in series with the battery. The voltmeter is connected to the terminals of the lamp. The graph obtained will not be a straight line owing to the heating of the filament by the current.)

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